SOCIETY OF ACTUARIES

EXAM STAM SHORT-TERM ACTUARIAL MATHEMATICS

EXAM STAM SAMPLE SOLUTIONS

Questions 1-307 have been taken from the previous set of Exam C sample questions. Questions no longer relevant to the syllabus have been deleted. Question 308-326 are based on material newly added.

April 2018 update: Question 303 has been deleted. Corrections were made to several of the new questions, 308-326.

December 2018 update: Corrections were made to questions 322, 323, and 325. Questions 327 and 328 were added.

Some of the questions in this study note are taken from past examinations. The weight of topics in these sample questions is not representative of the weight of topics on the exam. The syllabus indicates the exam weights by topic.

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Question #1 - DELETED

Question #2
Key: E

The standard for full credibility is $\left(\frac{1.645}{0.02}\right)^2\left(1+\frac{Var(X)}{E(X)^2}\right)$ where X is the claim size variable.

For the Pareto variable, $E(X)=0.5/5=0.1$ and $Var(X)=\frac{2(0.5)^2}{5(4)}-(0.1)^2=0.015$. Then the standard is $\left(\frac{1.645}{0.02}\right)^2\left(1+\frac{0.015}{0.1^2}\right)=16,913$ claims.

Question #3 - DELETED

Question #4
Key: A

The distribution function is $F(x)=\int_1^x \alpha t^{-\alpha-1}dt=-t^{-\alpha}\Big|_1^x=1-x^{-\alpha}$. The likelihood function is

$$\begin{aligned}L&=f(3)f(6)f(14)[1-F(25)]^2\\&=\alpha 3^{-\alpha-1}\alpha 6^{-\alpha-1}\alpha 14^{-\alpha-1}(25^{-\alpha})^2\\&\propto \alpha^3[3(6)(14)(625)]^{-\alpha}.\end{aligned}$$

Taking logs, differentiating, setting equal to zero, and solving:

$\ln L=3\ln\alpha-\alpha\ln 157,500$ plus a constant

$d\ln L/d\alpha=3\alpha^{-1}-\ln 157,500=0$

$\hat{\alpha}=3/\ln 157,500=0.2507.$

Question #5
Key: C

$\pi(q\,|\,1,1)\propto p(1\,|\,q)p(1\,|\,q)\pi(q)=2q(1-q)2q(1-q)4q^3\propto q^5(1-q)^2$

$\int_0^1 q^5(1-q)^2dq=1/168,\ \pi(q\,|\,1,1)=168q^5(1-q)^2.$

The expected number of claims in a year is $E(X\,|\,q)=2q$ and so the Bayesian estimate is

$E(2q\,|\,1,1)=\int_0^1 2q(168)q^5(1-q)^2dq=4/3.$

The answer can be obtained without integrals by recognizing that the posterior distribution of q is beta with a = 6 and b = 3. The posterior mean is $E(q\,|\,1,1)=a/(a+b)=6/9=2/3$ The posterior mean of $2q$ is then 4/3.

Question #6 - DELETED

Question #7 - DELETED

Question #8
Key: C

Let N be the Poisson claim count variable, let X be the claim size variable, and let S be the aggregate loss variable.

$$\mu(\theta) = E(S \mid \theta) = E(N \mid \theta)E(X \mid \theta) = \theta 10\theta = 10\theta^2$$

$$v(\theta) = Var(S \mid \theta) = E(N \mid \theta)E(X^2 \mid \theta) = \theta 200\theta^2 = 200\theta^3$$

$$\mu = E(10\theta^2) = \int_1^\infty 10\theta^2(5\theta^{-6})d\theta = 50/3$$

$$EPV = E(200\theta^3) = \int_1^\infty 200\theta^3(5\theta^{-6})d\theta = 500$$

$$VHM = Var(10\theta^2) = \int_1^\infty (10\theta^2)^2(5\theta^{-6})d\theta - (50/3)^2 = 222.22$$

$$k = 500/222.22 = 2.25.$$

Question #9- DELETED

Question #10 - DELETED

Question #11
Key: D

$$\Pr(\theta = 1 \mid X = 5) = \frac{f(5 \mid \theta = 1)\Pr(\theta = 1)}{f(5 \mid \theta = 1)\Pr(\theta = 1) + f(5 \mid \theta = 3)\Pr(\theta = 3)}$$

$$= \frac{(1/36)(1/2)}{(1/36)(1/2) + (3/64)(1/2)} = 16/43$$

$$\Pr(X_2 > 8 \mid X_1 = 5) = \Pr(X_2 > 8 \mid \theta = 1)\Pr(\theta = 1 \mid X_1 = 5) + \Pr(X_2 > 8 \mid \theta = 3)\Pr(\theta = 3 \mid X_1 = 5)$$

$$= (1/9)(16/43) + (3/11)(27/43) = 0.2126.$$

For the last line, $\Pr(X > 8 \mid \theta) = \int_8^\infty \theta(x+\theta)^{-2}dx = \theta(8+\theta)^{-1}$ is used.

Question #12
Key: C

The sample mean for X is 720 and for Y is 670. The mean of all 8 observations is 695.

$$\hat{v} = [(730-720)^2 + \cdots + (700-720)^2 + (655-670)^2 + \cdots + (750-670)^2]/[2(4-1)] = 3475$$

$$\hat{a} = [(720-695)^2 + (670-695)^2]/(2-1) - 3475/4 = 381.25$$

$$\hat{k} = 3475/381.25 = 9.1148$$

$$\hat{Z} = 4/(4+9.1148) = 0.305$$

$$P_C = 0.305(670) + 0.695(695) = 687.4.$$

Question #13
Key: B

There are 430 observations. The expected counts are 430(0.2744) = 117.99, 430(0.3512) = 151.02, and 430(0.3744) = 160.99. The test statistic is

$$\frac{(112-117.99)^2}{117.99} + \frac{(180-151.02)^2}{151.02} + \frac{(138-160.99)^2}{160.99} = 9.15.$$

Question #14
Key: B

From the information, the asymptotic variance of $\hat{\theta}$ is $1/4n$. Then

$$Var(2\hat{\theta}) = 4Var(\hat{\theta}) = 4(1/4n) = 1/n$$

Question #15
Key: A

$$\pi(p\,|\,1,1,1,1,1,1,1,1) \propto \Pr(1,1,1,1,1,1,1,1\,|\,p)\pi(p) \propto p^8$$

$$\pi(p\,|\,1,1,1,1,1,1,1,1) = \frac{p^8}{\int_0^{0.5} p^8 dp} = \frac{p^8}{0.5^9/9} = 9(0.5^{-9})p^8$$

$$\Pr(X_9 = 1\,|\,1,1,1,1,1,1,1,1) = \int_0^{0.5} \Pr(X_9 = 1\,|\,p)\pi(p\,|\,1,1,1,1,1,1,1,1)dp$$

$$= \int_0^{0.5} p9(0.5^{-9})p^8 dp = 9(0.5^{-9})(0.5^{10})/10 = 0.45.$$

Question #16 - DELETED

Question #17 - DELETED

Question #18
Key: D

The means are 0.5(250) + 0.3(2,500) + 0.2(60,000) = 12,875 and 0.7(250) + 0.2(2,500) + 0.1(60,000) = 6,675 for risks 1 and 2 respectively.
The variances are $0.5(250)^2 + 0.3(2{,}500)^2 + 0.2(60{,}000)^2 - 12{,}875^2 = 556{,}140{,}625$ and $0.7(250)^2 + 0.2(2{,}500)^2 + 0.1(60{,}000)^2 - 6{,}675^2 = 316{,}738{,}125$ respectively.

The overall mean is (2/3)(12,875) + (1/3)(6,675) = 10,808.33 and so
EPV = (2/3)(556,140,625) + (1/3)(316,738,125) = 476,339,792 and
VHM = $(2/3)(12{,}875)^2 + (1/3)(6{,}675)^2 - 10{,}808.33^2 = 8{,}542{,}222$. Then,
k = 476,339,792/8,542,222 = 55.763 and Z = 1/(1 + 55.763) = .017617.
The credibility estimate is .017617(250) + .982383(10,808.33) = 10,622.

Question #19 - DELETED

Question #20 - DELETED

Question #21
Key: B

From the Poisson distribution, $\mu(\lambda) = \lambda$ and $v(\lambda) = \lambda$. Then,
$\mu = E(\lambda) = 6/100 = 0.06, EPV = E(\lambda) = 0.06, VHM = Var(\lambda) = 6/100^2 = 0.0006$ where the various moments are evaluated from the gamma distribution. Then, $k = 0.06/0.0006 = 100$ and Z = 450/(450 + 100) = 9/11 where 450 is the total number of insureds contributing experience. The credibility estimate of the expected number of claims for one insured in month 4 is (9/11)(25/450) + (2/11)(0.06) = 0.056364. For 300 insureds the expected number of claims is 300(0.056364) = 16.9.

Question #22
Key: C

The likelihood function is $L(\alpha,\theta) = \prod_{j=1}^{200} \frac{\alpha\theta^{\alpha}}{(x_j + \theta)^{\alpha+1}}$ and its logarithm is
$l(\alpha,\theta) = 200\ln(\alpha) + 200\alpha\ln(\theta) - (\alpha+1)\sum_{j=1}^{200}\ln(x_j + \theta)$. When evaluated at the hypothesized values of 1.5 and 7.8, the loglikelihood is –821.77. The test statistic is 2(821.77 – 817.92) = 7.7. With two degrees of freedom (0 free parameters in the null hypothesis versus 2 in the alternative), the test statistic falls between the 97.5th percentile (7.38) and the 99th percentile (9.21).

Question #23
Key: E

Assume that $\theta > 5$. Then the expected counts for the three intervals are $15(2/\theta) = 30/\theta, 15(3/\theta) = 45/\theta$, and $15(\theta - 5)/\theta = 15 - 75/\theta$ respectively. The quantity to minimize is

$$\frac{1}{5}[(30\theta^{-1} - 5)^2 + (45\theta^{-1} - 5)^2 + (15 - 75\theta^{-1} - 5)^2].$$

Differentiating (and ignoring the coefficient of 1/5) gives the equation $-2(30\theta^{-1} - 5)30\theta^{-2} - 2(45\theta^{-1} - 5)45\theta^{-2} + 2(10 - 75\theta^{-1})75\theta^{-2} = 0.$ Multiplying through by θ^3 and dividing by 2 reduces the equation to $-(30 - 5\theta)30 - (45 - 5\theta)45 + (10\theta - 75)75 = -8550 + 1125\theta = 0$ for a solution of $\hat{\theta} = 8550/1125 = 7.6.$

Question #24
Key: E

$\pi(\theta | 1) \propto \theta(1.5\theta^{0.5}) \propto \theta^{1.5}$. The required constant is the reciprocal of $\int_0^1 \theta^{1.5} d\theta = 0.4$ and so $\pi(\theta | 1) = 2.5\theta^{1.5}$. The requested probability is

$$\Pr(\theta > 0.6 | 1) = \int_{0.6}^1 2.5\theta^{1.5} d\theta = \theta^{2.5}\Big|_{0.6}^1 = 1 - 0.6^{2.5} = 0.721.$$

Question #25
Key: A

k	kn_k / n_{k-1}
0	
1	0.81
2	0.92
3	1.75
4	2.29
5	2.50
6	3.00

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #26
Key: B

The likelihood function is $\frac{e^{-1/(2\theta)}}{2\theta}\frac{e^{-2/(2\theta)}}{2\theta}\frac{e^{-3/(2\theta)}}{2\theta}\frac{e^{-15/(3\theta)}}{3\theta}=\frac{e^{-8/\theta}}{24\theta^4}$. The loglikelihood function is $-\ln(24)-4\ln(\theta)-8/\theta$. Differentiating with respect to θ and setting the result equal to 0 yields $-\frac{4}{\theta}+\frac{8}{\theta^2}=0$ which produces $\hat{\theta}=2$.

Question #27
Key: E

The absolute difference of the credibility estimate from its expected value is to be less than or equal to $k\mu$ (with probability *P*). That is,

$$\left|[ZX_{partial}+(1-Z)M]-[Z\mu+(1-Z)M]\right|\le k\mu$$

$$-k\mu\le ZX_{partial}-Z\mu\le k\mu.$$

Adding μ to all three sides produces answer choice (E).

Question #28
Key: C

In general,

$$E(X^2)-E[(X\wedge 150)^2]=\int_0^{200}x^2f(x)dx-\int_0^{150}x^2f(x)dx-150^2\int_{150}^{200}f(x)dx$$

$$=\int_{150}^{200}(x^2-150^2)f(x)dx.$$

Assuming a uniform distribution, the density function over the interval from 100 to 200 is 6/7400 (the probability of 6/74 assigned to the interval divided by the width of the interval). The answer is

$$\int_{150}^{200}(x^2-150^2)\frac{6}{7400}dx=\left(\frac{x^3}{3}-150^2x\right)\frac{6}{7400}\Bigg|_{150}^{200}=337.84.$$

Question #29
Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

$$\Pr(3 \mid I) = \binom{6}{3}(0.1)^3(0.9)^3 = 0.01458$$

$$\Pr(3 \mid II) = \binom{6}{3}(0.2)^3(0.8)^3 = 0.08192$$

$$\Pr(3 \mid III) = \binom{6}{3}(0.4)^3(0.6)^3 = 0.27648$$

The probability of observing three successes is 0.7(.01458) + 0.2(.08192) + 0.1(.27648) = 0.054238. The three posterior probabilities are:

$$\Pr(I \mid 3) = \frac{0.7(0.01458)}{0.054238} = 0.18817$$

$$\Pr(II \mid 3) = \frac{0.2(0.08192)}{0.054238} = 0.30208$$

$$\Pr(III \mid 3) = \frac{0.1(0.27648)}{0.054238} = 0.50975.$$

The posterior probability of a claim is then 0.1(0.18817) + 0.2(0.30208) + 0.4(0.50975) = 0.28313.

Question #30 - DELETED

Question # 31 - DELETED

Question # 32
Key: D

N is distributed Poisson(λ)

$$\mu = E(\lambda) = \alpha\theta = 1(1.2) = 1.2$$

$$v = E(\lambda) = 1.2,\ a = Var(\lambda) = \alpha\theta^2 = 1(1.2)^2 = 1.44$$

$$k = \frac{1.2}{1.44} = \frac{5}{6},\ Z = \frac{2}{2 + 5/6} = \frac{12}{17}$$

Thus, the estimate for Year 3 is $\frac{12}{17}(1.5) + \frac{5}{17}(1.2) = 1.41.$

Note that a Bayesian approach produces the same answer.

Question # 33 - DELETED

Question # 34
Key: B

The likelihood is:

$$L=\prod_{j=1}^{n}\frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}}\propto\prod_{j=1}^{n}\beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l=\sum_{j=1}^{n}[x_j\ln\beta-(r+x_j)\ln(1+\beta)]$$
$$l'=\sum_{j=1}^{n}\left[\frac{x_j}{\beta}-\frac{r+x_j}{1+\beta}\right]=0$$
$$0=\sum_{j=1}^{n}[x_j(1+\beta)-(r+x_j)\beta]=\sum_{j=1}^{n}x_j-rn\beta=n\bar{x}-rn\beta$$
$$\hat{\beta}=\bar{x}/r.$$

Question # 35
Key: C

The Bühlmann credibility estimate is $Zx+(1-Z)\mu$ where x is the first observation. The Bühlmann estimate is the least squares approximation to the Bayesian estimate. Therefore, Z and μ must be selected to minimize

$$\frac{1}{3}[Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[2Z+(1-Z)\mu-1.5]^2+\frac{1}{3}[3Z+(1-Z)\mu-3]^2.$$

Setting partial derivatives equal to zero will give the values. However, it should be clear that μ is the average of the Bayesian estimates, that is,

$$\mu=\frac{1}{3}(1.5+1.5+3)=2.$$

The derivative with respect to Z is (deleting the coefficients of 1/3):

$$2(-Z+0.5)(-1)+2(0.5)(0)+2(Z-1)(1)=0$$
$$4Z-3=0,\ Z=0.75.$$

The answer is 0.75(1) + 0.25(2) = 1.25.

Question # 36 - DELETED

Question # 37
Key: B

The likelihood is:

$$L=\frac{\alpha 150^{\alpha}}{(150+225)^{\alpha+1}}\frac{\alpha 150^{\alpha}}{(150+525)^{\alpha+1}}\frac{\alpha 150^{\alpha}}{(150+950)^{\alpha+1}}=\frac{\alpha^{3}150^{3\alpha}}{[(375)(675)(1100)]^{\alpha+1}}.$$

The loglikelihood is:

$$l=3\ln\alpha+3\alpha\ln 150-(\alpha+1)\ln[(375)(675)(1100)]$$
$$l'=3\alpha^{-1}+3\ln 150-\ln[(375)(675)(1100)]=3\alpha^{-1}-4.4128$$
$$\hat{\alpha}=3/4.4128=0.6798.$$

Question # 38
Key: D

For this problem, r = 4 and n = 7. Then,

$$\hat{v}=\frac{33.60}{4(7-1)}=1.4,\ \hat{a}=\frac{3.3}{4-1}-\frac{1.4}{7}=0.9.$$

Then,

$$k=\frac{1.4}{0.9}=\frac{14}{9},\ Z=\frac{7}{7+(14/9)}=\frac{63}{77}=0.82.$$

Question # 39
Key: B

X is the random sum $Y_1+Y_2+\cdots+Y_N$.
N has a negative binomial distribution with r = a = 1.5 and $\beta=\theta=0.2$.

$$E(N)=r\beta=0.3, Var(N)=r\beta(1+\beta)=0.36$$
$$E(Y)=5{,}000, Var(Y)=25{,}000{,}000$$
$$E(X)=0.3(5{,}000)=1{,}500$$
$$Var(X)=0.3(25{,}000{,}000)+0.36(25{,}000{,}000)=16{,}500{,}000$$

Number of exposures (insureds) required for full credibility
$n_{FULL}=(1.645/0.05)^2(16{,}500{,}000/1{,}500^2)=7{,}937.67.$

Number of expected claims required for full credibility
$E(N)n_{FULL}=0.3(7{,}937.67)=2{,}381.$

Question # 40
Key: E

X	$F_n(x)$	$F_n(x^-)$	$F_0(x)$	$\|F_n(x)-F_0(x)\|$	$\|F_n(x^-)-F_0(x)\|$
29	0.2	0	0.252	0.052	0.252
64	0.4	0.2	0.473	0.073	0.273
90	0.6	0.4	0.593	0.007	0.193
135	0.8	0.6	0.741	0.059	0.141
182	1.00	0.8	0.838	0.162	0.038

where:
$\hat{\theta} = \bar{x} - 100$ and $F_0(x) = 1 - e^{-x/100}$.
The maximum value from the last two columns is 0.273.

Question # 41
Key: E

$\mu = E(\lambda) = 1, v = E(\sigma^2) = 1.25, a = Var(\lambda) = 1/12$
$k = v/a = 15, Z = 1/(1+15) = 1/16.$

Thus, the estimate for Year 2 is (1/16)(0) + (15/16)(1) = 0.9375.

Question # 42 - DELETED

Question # 43
Key: E

The posterior density, given an observation of 3 is:

$$\pi(\theta \mid 3) = \frac{f(3\mid\theta)\pi(\theta)}{\int_1^\infty f(3\mid\theta)\pi(\theta)d\theta} = \frac{\frac{2\theta^2}{(3+\theta)^3}\frac{1}{\theta^2}}{\int_1^\infty 2(3+\theta)^{-3}d\theta} = \frac{2(3+\theta)^{-3}}{-(3+\theta)^{-2}\Big|_1^\infty} = 32(3+\theta)^{-3}, \theta > 1.$$

Then,

$$\Pr(\Theta > 2) = \int_2^\infty 32(3+\theta)^{-3}d\theta = -16(3+\theta)^{-2}\Big|_2^\infty = 16/25 = 0.64.$$

Question # 44
Key: B

$$L = F(1000)^7[F(2000) - F(1000)]^6[1 - F(2000)]^7$$
$$= (1 - e^{-1000/\theta})^7(e^{-1000/\theta} - e^{-2000/\theta})^6(e^{-2000/\theta})^7$$
$$= (1-p)^7(p-p^2)^6(p^2)^7 = p^{20}(1-p)^{13}$$

where $p = e^{-1000/\theta}$ The maximum occurs at p = 20/33 and so $\hat{\theta} = -1000\ln(20/33) = 1996.90$.

Question # 45
Key: A

$$E(X \mid \theta) = \theta / 2$$
$$E(X_3 \mid 400, 600) = \int_{600}^{\infty} E(X \mid \theta) f(\theta \mid 400, 600) d\theta = \int_{600}^{\infty} \frac{\theta}{2} 3 \frac{600^3}{\theta^4} d\theta$$
$$= \frac{3(600^3)}{2} \frac{\theta^{-2}}{-2} \bigg|_{600}^{\infty} = \frac{3(600^3)(600^{-2})}{4} = 450.$$

Question # 46 - DELETED

Question # 47
Key: C

The maximum likelihood estimate for the Poisson distribution is the sample mean:

$$\hat{\lambda} = \overline{x} = \frac{50(0)+122(1)+101(2)+92(3)}{365} = 1.6438.$$

The table for the chi-square test is:

Number of days	Probability	Expected*	Chi-square
0	$e^{-1.6438} = 0.19324$	70.53	5.98
1	$1.6438e^{-1.6438} = 0.31765$	115.94	0.32
2	$\frac{1.6438^2 e^{-1.6438}}{2} = 0.26108$	95.30	0.34
3+	0.22803**	83.23	0.92

*365x(Probability) **obtained by subtracting the other probabilities from 1

The sum of the last column is the test statistic of 7.56. Using 2 degrees of freedom (4 rows less 1 estimated parameter less 1) the model is rejected at the 2.5% significance level but not at the 1% significance level.

Question # 48
Key: D

$$\mu(0) = \frac{0.4(0)+0.1(1)+0.1(2)}{0.6} = 0.5,\ \mu(1) = \frac{0.1(0)+0.2(1)+0.1(2)}{0.4} = 1$$

$$\mu = 0.6(0.5)+0.4(1) = 0.7$$

$$a = 0.6(0.5^2)+0.4(1^2)-0.7^2 = 0.06$$

$$v(0) = \frac{0.4(0)+0.1(1)+0.1(4)}{0.6} - 0.5^2 = 7/12,\ v(1) = \frac{0.1(0)+0.2(2)+0.1(4)}{0.4} - 1^2 = 0.5$$

$$v = 0.6(7/12)+0.4(0.5) = 11/20$$

$$k = v/a = (11/20)/0.06 = 11/1.2 = 55/6,\ Z = \frac{10}{10+55/6} = 60/115 = 12/23$$

Bühlmann credibility premium = $\frac{12}{23}\frac{10}{10} + \frac{11}{23}(0.7) = 0.8565..$

Question # 49 - DELETED

Question # 50
Key: C

The four classes have means 0.1, 0.2, 0.5, and 0.9 respectively and variances 0.09, 0.16, 0.25, and 0.09 respectively.

Then,

$$\mu = 0.25(0.1 + 0.2 + 0.5 + 0.9) = 0.425$$

$$v = 0.25(0.09 + 0.16 + 0.25 + 0.09) = 0.1475$$

$$a = 0.25(0.010.04 + 0.25 + 0.81) - 0.425^2 = 0.096875$$

$$k = 0.1475 / 0.096875 = 1.52258$$

$$Z = \frac{4}{4 + 1.52258} = 0.7243$$

The estimate is 5[0.7243(2/4) + 0.2757(0.425)] = 2.40.

Question # 51 - DELETED

Question # 52 - DELETED

Question # 53
Key: B

First obtain the distribution of aggregate losses:

Value	Probability
0	1/5
25	(3/5)(1/3) = 1/5
100	(1/5)(2/3)(2/3) = 4/45
150	(3/5)(2/3) = 2/5
250	(1/5)(2)(2/3)(1/3) = 4/45
400	(1/5)(1/3)(1/3) = 1/45

$$\mu = (1/5)(0) + (1/5)(25) + (4/45)(100) + (2/5)(150) + (4/45)(250) + (1/45)(400) = 105$$

$$\sigma^2 = (1/5)(0^2) + (1/5)(25) + (4/45)(100^2) + (2/5)(150^2) + (4/45)(250^2)$$
$$+ (1/45)(400^2) - 105^2 = 8100$$

Question # 54 - DELETED

Question # 55
Key: B

$$\Pr(class1|1) = \frac{(1/2)(1/3)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{3}{4}$$
$$\Pr(class2|1) = \frac{(1/3)(1/6)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = \frac{1}{4}$$
$$\Pr(class3|1) = \frac{(1/6)(0)}{(1/2)(1/3)+(1/3)(1/6)+(1/6)(0)} = 0$$

because the prior probabilities for the three classes are 1/2, 1/3, and 1/6 respectively.

The class means are

$$\mu(1) = (1/3)(0)+(1/3)(1)+(1/3)(2) = 1$$
$$\mu(2) = (1/6)(1)+(2/3)(2)+(1/6)(3) = 2$$

The expectation is

$$E(X_2|1) = (3/4)(1)+(1/4)(2) = 1.25.$$

Question # 56
Key: E

The first, second, third, and sixth payments were observed at their actual value and each contributes $f(x)$ to the likelihood function. The fourth and fifth payments were paid at the policy limit and each contributes $1 - F(x)$ to the likelihood function. This is answer (E).

Question #57 - DELETED

Question #58
Key: B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $\mu(\lambda) = \mathrm{v}(\lambda) = \lambda$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$$\mu = v = E(\lambda) = 50(1/500) = 0.1$$
$$a = Var(\lambda) = 50(1/500)^2 = 0.0002$$
$$k = v/a = 500$$
$$Z = 1500/(1500+500) = 0.75$$
$$\bar{x} = (75+210)/(600+900) = 0.19$$

The credibility estimate is 0.75(0.19) + 0.25(0.1) = 0.1675. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter λ times the number of policies) $\frac{e^{-600\lambda}(600\lambda)^{75}}{75!}\frac{e^{-900\lambda}(900\lambda)^{210}}{210!}\frac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)} \propto \lambda^{105}e^{-2000\lambda}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.

uestion #59
Key: E

The *q*-*q* plot takes the ordered values and plots the *j*th point at *j*/(*n*+1) on the horizontal axis and at $F(x_j;\theta)$ on the vertical axis. For small values, the model assigns more probability to being below that value than occurred in the sample. This indicates that the model has a heavier left tail than the data. For large values, the model again assigns more probability to being below that value (and so less probability to being above that value). This indicates that the model has a lighter right tail than the data. Of the five answer choices, only E is consistent with these observations. In addition, note that as you go from 0.4 to 0.6 on the horizontal axis (thus looking at the middle 20% of the data), the *q*-*q* plot increases from about 0.3 to 0.4 indicating that the model puts only about 10% of the probability in this range, thus confirming answer E.

Question #60
Key: C

The posterior probability of having one of the coins with a 50% probability of heads is proportional to (0.5)(0.5)(0.5)(0.5)(4/6) = 0.04167. This is obtained by multiplying the probabilities of making the successive observations 1, 1, 0, and 1 with the 50% coin times the prior probability of 4/6 of selecting this coin. The posterior probability for the 25% coin is proportional to (0.25)(0.25)(0.75)(0.25)(1/6) = 0.00195 and the posterior probability for the 75% coin is proportional to (0.75)(0.75)(0.25)(0.75)(1/6) = 0.01758. These three numbers total 0.06120. Dividing by this sum gives the actual posterior probabilities of 0.68088, 0.03186, and 0.28726. The expected value for the fifth toss is then (.68088)(0.5) + (.03186)(0.25) + (.28726)(0.75) = 0.56385.

Question #61
Key: A

Because the exponential distribution is memoryless, the excess over the deductible is also exponential with the same parameter. So subtracting 100 from each observation yields data from an exponential distribution and noting that the maximum likelihood estimate is the sample mean gives the answer of 73.

Working from first principles,

$$L(\theta)=\frac{f(x_1)f(x_2)f(x_3)f(x_4)f(x_5)}{[1-F(100)]^5}=\frac{\theta^{-1}e^{-125/\theta}\theta^{-1}e^{-150/\theta}\theta^{-1}e^{-165/\theta}\theta^{-1}e^{-175/\theta}\theta^{-1}e^{-250/\theta}}{(e^{-100/\theta})^5}$$
$$=\theta^{-5}e^{-365/\theta}.$$

Taking logarithms and then a derivative gives

$$l(\theta)=-5\ln(\theta)-365/\theta,\ l'(\theta)=-5/\theta+365/\theta^2=0$$
$$\hat{\theta}=365/5=73.$$

Question #62
Key: D

The number of claims for each insured has a binomial distribution with n = 1 and q unknown. We have

$$\mu(q) = q, v(\theta) = q(1-q)$$
$$\mu = E(q) = 0.1 \text{ (see iv)}$$
$$a = Var(q) = E(q^2) - 0.1^2 = 0.01 \text{ (see v)} \Rightarrow E(q^2) = 0.02$$
$$v = E[q(1-q)] = E(q) - E(q^2) = 0.1 - 0.02 = 0.08$$
$$k = v / a = 8, Z = 10 / (10+8) = 5/9$$

Then the expected number of claims in the next one year is (5/9)(0) + (4/9)(0.1) = 2/45 and the expected number of claims in the next five years is 5(2/45) = 2/9 = 0.22.

Question #63 - DELETED

Question #64
Key: E

The model distribution is $f(x|\theta) = 1/\theta, 0 < x < \theta$. Then the posterior distribution is proportional to

$$\pi(\theta | 400,600) \propto \frac{1}{\theta}\frac{1}{\theta}\frac{500}{\theta^2} \propto \theta^{-4}, \theta > 600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible parameter values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} \theta^{-4} d\theta = \frac{1}{3(600)^3}$ and thus the exact posterior density is $\pi(\theta | 400,600) = 3(600)^3 \theta^{-4}, \theta > 600$. The posterior probability of an observation exceeding 550 is

$$\Pr(X_3 > 550 | 400,600) = \int_{600}^{\infty} \Pr(X_3 > 550 | \theta)\pi(\theta | 400,600) d\theta = \int_{600}^{\infty} \frac{\theta - 550}{\theta} 3(600)^3 \theta^{-4} d\theta = 0.3125.$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.

Question #65
Key: C

$$E(N) = r\beta = 0.4$$
$$Var(N) = r\beta(1+\beta) = 0.48$$
$$E(Y) = \theta / (\alpha - 1) = 500$$
$$Var(Y) = \frac{\theta^2 \alpha}{(\alpha-1)^2(\alpha-2)} = 750,000$$

Therefore,
$$E(X) = 0.4(500) = 200$$
$$Var(X) = 0.4(750,000) + 0.48(500)^2 = 420,000.$$

The full credibility standard is $n = \left(\frac{1.645}{0.05}\right)^2 \frac{420,000}{200^2} = 11,365,\ Z = \sqrt{2,500/11,365} = 0.47.$

Question #66 - DELETED

Question #67
Key: E

$$\mu(r) = E(X \mid r) = E(N)E(Y) = r\beta\theta / (\alpha - 1) = 100r$$
$$v(r) = Var(X \mid r) = Var(N)E(Y)^2 + E(N)Var(Y)$$
$$= \frac{r\beta(1+\beta)\theta^2}{(\alpha-1)^2} + \frac{r\beta\alpha\theta^2}{(\alpha-1)^2(\alpha-2)} = 210,000r$$
$$v = E(210,000r) = 210,000(2) = 420,000$$
$$a = Var(100r) = (100)^2(4) = 40,000$$
$$k = v/a = 10.5,\ Z = 100/(100+10.5) = 0.905$$

Question #68 - DELETED

Question #69
Key: B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have

$$E(\bar{X}) = E(X) = \theta, Var(\bar{X}) = Var(X) / n = \theta^2 / n$$

$$cv = SD(\bar{X}) / E(\bar{X}) = (\theta / \sqrt{n}) / \theta = 1 / \sqrt{n} = 1 - \sqrt{5} = 0.447.$$

If the maximum likelihood estimator is not known, it can be derived:

$$L(\theta) = \theta^{-n} e^{-\Sigma x/\theta}, l(\theta) = -n \ln \theta - n\bar{X} / \theta, l'(\theta) = -n\theta^{-1} + n\bar{X}\theta^{-2} = 0 \Rightarrow \hat{\theta} = \bar{X}.$$

Question #70
Key: D

Because the total expected claims for business use is 1.8, it must be that 20% of business users are rural and 80% are urban. Thus the unconditional probabilities of being business-rural and business-urban are 0.1 and 0.4 respectively. Similarly the probabilities of being pleasure-rural and pleasure-urban are also 0.1 and 0.4 respectively. Then,

$$\mu = 0.1(1.0) + 0.4(2.0) + 0.1(1.5) + 0.4(2.5) = 2.05$$

$$v = 0.1(0.5) + 0.4(1.0) + 0.1(0.8) + 0.4(1.0) = 0.93$$

$$a = 0.1(1.0)^2 + 0.4(2.0)^2 + 0.1(1.5)^2 + 0.4(2.5)^2 - 2.05^2 = 0.2225$$

$$k = v / a = 4.18, Z = 1 / (1 + 4.18) = 0.193.$$

Question #71
Key: A

No. claims	Hypothesized	Observed	Chi-square
1	250	235	15(15)/250 = 0.90
2	350	335	15(15)/350 = 0.64
3	240	250	10(10)/240 = 0.42
4	110	111	1(1)/110 = 0.01
5	40	47	7(7)/40 = 1.23
6+	10	22	12(12)/10 = 14.40

The last column sums to the test statistic of 17.60 with 5 degrees of freedom (there were no estimated parameters), so from the table reject at the 0.005 significance level.

Question #72
Key: C

In part (ii) you are given that $\mu = 20$. In part (iii) you are given that *a* = 40. In part (iv) you are given that *v* = 8,000. Therefore, *k* = *v*/*a* = 200. Then,

$$\bar{X} = \frac{800(15)+600(10)+400(5)}{1800} = \frac{100}{9}$$

$$Z = \frac{1800}{1800+200} = 0.9$$

$$P_C = 0.9(100/9)+0.1(20) = 12.$$

Question #73 - DELETED

Question #74 - DELETED

Question #75 - DELETED

Question #76
Key: D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $\pi(\theta \mid N>0) \propto \Pr(N>0 \mid \theta)\pi(\theta) = (1-e^{-\theta})\theta e^{-\theta}$.

The constant of proportionality is obtained from $\int_0^\infty \theta e^{-\theta} - \theta e^{-2\theta} d\theta = \frac{1}{1^2} - \frac{1}{2^2} = 0.75.$
The posterior density is $\pi(\theta \mid N>0) = (1/0.75)(\theta e^{-\theta} - \theta e^{-2\theta})$.

Then,

$$\Pr(N_2>0 \mid N_1>0) = \int_0^\infty \Pr(N_2>0 \mid \theta)\pi(\theta \mid N_1>0)d\theta = \int_0^\infty (1-e^{-\theta})(4/3)(\theta e^{-\theta} - \theta e^{-2\theta})d\theta$$

$$= \frac{4}{3}\int_0^\infty \theta e^{-\theta} - 2\theta e^{-2\theta} + \theta e^{-3\theta} d\theta = \frac{4}{3}\left(\frac{1}{1^2} + \frac{2}{2^2} + \frac{1}{3^2}\right) = 0.8148.$$

Question #77 - DELETED

Question #78
Key: B

From item (ii), $\mu = 1000$ and *a* = 50. From item (i), *v* = 500. Therefore, *k* = *v*/*a* = 10 and *Z* = 3/(3+10) = 3/13. Also, $\bar{X} = (750+1075+2000)/3 = 1275$. Then
$P_c = (3/13)(1275)+(10/13)(1000) = 1063.46.$

Question #79
Key: C

$$f(x) = p\frac{1}{100}e^{-x/100} + (1-p)\frac{1}{10,000}e^{-x/10,000}$$

$$L(100,200) = f(100)f(2000)$$

$$= \left(\frac{pe^{-1}}{100} + \frac{(1-p)e^{-0.01}}{10,000}\right)\left(\frac{pe^{-20}}{100} + \frac{(1-p)e^{-0.2}}{10,000}\right)$$

Question #80 - DELETED

Question # 81 - DELETED

Question #82 - DELETED

Question #83 - DELETED

Question #84
Key: A

$$B = \begin{cases} c(400-x) & x < 400 \\ 0 & x \geq 400 \end{cases}$$

$$100 = E(B) = c400 - cE(X \wedge 400)$$

$$= c400 - c300\left(1 - \frac{300}{300+400}\right)$$

$$= c\left(400 - 300\frac{4}{7}\right)$$

$$c = \frac{100}{228.6} = 0.44$$

Question #85
Key: C

Let N = number of computers in department
Let X = cost of a maintenance call
Let S = aggregate cost

$$\text{Var}(X) = [\text{Standard Deviation}(X)]^2 = 200^2 = 40{,}000$$
$$E(X^2) = \text{Var}(X) + [E(X)]^2$$
$$= 40{,}000 + 80^2 = 46{,}400$$
$$E(S) = N\lambda E(X) = N(3)(80) = 240N$$
$$\text{Var}(S) = N\lambda \times E(X^2) = N(3)(46{,}400) = 139{,}200N$$

We want

$$0.1 \geq \Pr\left(S > 1.2E(S)\right)$$
$$\geq \Pr\left(\frac{S - E(S)}{\sqrt{139{,}200N}} > \frac{0.2E(S)}{\sqrt{139{,}200N}}\right) \Rightarrow \frac{0.2(240)N}{373.1\sqrt{N}} \geq 1.282 = \Phi(0.9)$$
$$N \geq \left(\frac{1.282(373.1)}{48}\right)^2 = 99.3$$

Question #86
Key: D

The modified severity, X^*, represents the conditional payment amount given that a payment occurs. Given that a payment is required ($X > d$), the payment must be uniformly distributed between 0 and $c(b - d)$.

The modified frequency, N^*, represents the number of losses that result in a payment. The deductible eliminates payments for losses below d, so only $1 - F_X(d) = \frac{b-d}{b}$ of losses will require payments. Therefore, the Poisson parameter for the modified frequency distribution is $\lambda \frac{b-d}{b}$. (Reimbursing c% after the deductible affects only the payment amount and not the frequency of payments).

Question #87
Key: E

$$f(x) = 0.01, \quad 0 \le x \le 80$$
$$= 0.01 - 0.00025(x-80) = 0.03 - 0.00025x, \quad 80 < x \le 120$$

$$E(x) = \int_0^{80} 0.01x\,dx + \int_{80}^{120} (0.03x - 0.00025x^2)dx$$
$$= \frac{0.01x^2}{2}\Big|_0^{80} + \frac{0.03x^2}{2}\Big|_{80}^{120} - \frac{0.00025x^3}{3}\Big|_{80}^{120}$$
$$= 32 + 120 - 101.33 = 50.66667$$

$$E(X-20)_+ = E(X) - \int_0^{20} x\,f(x)dx - 20\left[1 - \int_0^{20} f(x)dx\right]$$
$$= 50.6667 - \frac{0.01x^2}{2}\Big|_0^{20} - 20\left(1 - 0.01x\Big|_0^{20}\right)$$
$$= 50.6667 - 2 - 20(0.8) = 32.6667$$

Loss Elimination Ratio = $1 - \dfrac{32.6667}{50.6667} = 0.3553$

Question #88
Key: B

First restate the table to be CAC’s cost, after the 10% payment by the auto owner:

Towing Cost, x	$p(x)$
72	50%
90	40%
144	10%

Then $E(X) = 0.5(72) + 0.4(90) + 0.1(144) = 86.4$.

$$E(X^2) = 0.5(72^2) + 0.4(90^2) + 0.1(144^2) = 7905.6$$

$$\mathrm{Var}(X) = 7905.6 - 86.4^2 = 440.64$$

Because Poisson,

$$E(N) = Var(N) = 1000$$

$$E(S) = E(X)E(N) = 86.4(1000) = 86,400$$

$$\mathrm{Var}(S) = E(N)\mathrm{Var}(X) + E(X)^2\,\mathrm{Var}(N) = 1000(440.64) + 86.4^2(1000) = 7,905,600$$

$$\Pr(S > 90,000) + \Pr\left(\frac{S - E(S)}{\sqrt{\mathrm{Var}(S)}} > \frac{90,000 - 86,400}{\sqrt{7,905,600}}\right) = \Pr(Z > 1.28) = 1 - \Phi(1.28) = 0.10$$

Since the frequency is Poisson, you could also have used

$\mathrm{Var}(S) = \lambda E(X^2) = 1000(7905.6) = 7,905,600$.

That way, you would not need to have calculated Var(X).

Question #89
Key: C

$$\mathrm{LER} = \frac{E(X \wedge d)}{E(X)} = \frac{\theta(1 - e^{-d/\theta})}{\theta} = 1 - e^{-d/\theta}$$

Last year $\quad 0.70 = 1 - e^{-d/\theta} \Rightarrow -d = \theta \log(0.30)$

Next year: $\quad -d_{new} = \theta \log(1 - \mathrm{LER}_{new})$

Hence $\theta \log(1 - \mathrm{LER}_{\mathrm{new}}) = -d_{\mathrm{new}} = \frac{4}{3}\theta \log(0.30)$

$$\log\left(1 - \mathrm{LER}_{new}\right) = -1.6053$$

$$\left(1 - \mathrm{LER}_{new}\right) = e^{-1.6053} = 0.20$$

$$\mathrm{LER}_{new} = 0.80$$

Question # 90
Key: E

The distribution of claims (a gamma mixture of Poissons) is negative binomial.

$$E(N) = E_{\Lambda}[E(N \mid \Lambda)] = E_{\Lambda}(\Lambda) = 3$$
$$Var(N) = E_{\Lambda}[Var(N \mid \Lambda)] + Var_{\Lambda}[E(N \mid \Lambda)] = E_{\Lambda}(\Lambda) + Var_{\Lambda}(\Lambda) = 6$$
$$r\beta = 3$$
$$r\beta(1+\beta) = 6$$
$$(1+\beta) = 6/3 = 2;\ \beta = 1$$
$$r\beta = 3;\ r = 3$$

$$p_0 = (1+\beta)^{-r} = 0.125$$
$$p_1 = \frac{r\beta}{(1+\beta)^{r+1}} = 0.1875$$
$$\Pr(\text{at most } 1) = p_0 + p_1 = 0.3125.$$

Question # 91
Key: A

$$E(S) = E(N)E(X) = 110(1{,}101) = 121{,}110$$
$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 110(70^2) + 1101^2(750) = 909{,}689{,}750$$
$$StdDev(S) = 30{,}161$$
$$\Pr(S < 100{,}000) = \Pr\left(Z < \frac{100{,}000 - 121{,}110}{30{,}161} = -0.70\right) = 0.242$$

where Z has standard normal distribution.

Question # 92
Key: C

Let N = number of prescriptions then

n	$f_N(n)$	$F_N(n)$	$1-F_N(n)$
0	0.2000	0.2000	0.8000
1	0.1600	0.3600	0.6400
2	0.1280	0.4880	0.5120
3	0.1024	0.5904	0.4096

$$E(N) = 4 = \sum_{j=0}^{\infty}[1-F(j)]$$

$$E[(S-80)_+] = 40E[(N-2)_+] = 40\sum_{j=2}^{\infty}[1-F(j)]$$

$$= 40\left[\sum_{j=0}^{\infty}[1-F(j)] - \sum_{j=0}^{1}[1-F(j)]\right]$$

$$= 40(4-1.44) = 102.40$$

$$E[(S-120)_+] = 40E[(N-3)_+] = 40\left[\sum_{j=0}^{\infty}[1-F(j)] - \sum_{j=0}^{2}[1-F(j)]\right] = 40(4-1.952) = 81.92$$

Because no values of S between 80 and 120 are possible,

$$E[(S-100)_+] = \frac{(120-100)E[(S-80)_+] + (100-80)E[(S-120)_+]}{120-80} = 92.16$$

Alternatively,

$$E[(S-100)_+] = \sum_{j=0}^{\infty}(40j-100)f_N(j) + 100f_N(0) + 60f_N(1) + 20f_N(2)$$

(The correction terms are needed because (40j – 100) would be negative for j = 0, 1, 2; we need to add back the amount those terms would be negative)

$$= 40\sum_{j=0}^{\infty} jf_N(j) - 100\sum_{j=0}^{\infty} f_N(j) + 100(0.2) + 60(0.16) + 20(0.128)$$

$$= 40E(N) - 100 + 20 + 9.6 + 2.56 = 160 - 67.84 = 92.16$$

Question #93
Key: E

Method 1:

In each round,
N = result of first roll, to see how many dice you will roll
X = result of for one of the *N* dice you roll
S = sum of *X* for the *N* dice

$$E(X)=E(N)=3.5$$
$$Var(X)=Var(N)=2.9167$$
$$E(S)=E(N)E(X)=12.25$$
$$Var(S)=E(N)Var(X)+Var(N)E(X)^2=3.5(2.9167)+2.9167(3.5)^2=45.938$$

Let S_{1000} the sum of the winnings after 1000 rounds

$$E(S_{1000})=1000(12.25)=12,250$$
$$SD(S_{1000})=\sqrt{1000(45.938)}=214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings for a total of $2,500+S_{1000}$ Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$\Pr(2,500+S_{1000}>14,999.5)=\Pr(S_{1000}>12,499.5)\approx\Pr\left(Z>\frac{12,499.5-12,250}{214.33}=1.17\right)=0.12.$$

Question #94
Key: B

$$p_k=\left(a+\frac{b}{k}\right)p_{k-1}$$
$$0.25=(a+b)0.25\Rightarrow a=1-b$$
$$0.1875=\left(a+\frac{b}{2}\right)(0.25)\Rightarrow 0.1875=(1-0.5b)(0.25)\Rightarrow b=0.5,\ a=0.5$$
$$p_3=\left(0.5+\frac{0.5}{3}\right)(0.1875)=0.125$$

Question #95
Key: E

$\beta = \text{mean} = 4,\ p_k = \beta^k / (1+\beta)^{k+1}$

n	$P(N=n)$
0	0.2
1	0.16
2	0.128
3	0.1024

x	$f^{(1)}(x)$	$f^{(2)}(x)$	$f^{(3)}(x)$
0	0	0	0
1	0.25	0	0
2	0.25	0.0625	0
3	0.25	0.125	0.0156

$f_S(0) = 0.2,\ f_S(1) = 0.16(0.25) = 0.04$
$f_S(2) = 0.16(0.25) + 0.128(0.0625) = 0.049$
$f_S(3) = 0.16(0.25) + 0.128(0.125) + 0.1024(0.0156) = 0.0576$
$F_S(3) = 0.2 + 0.04 + 0.049 + 0.0576 = 0.346$

Question #96
Key: E

Let L = incurred losses; P = earned premium = 800,000
Bonus $0.15\left(0.6 - \frac{L}{P}\right)P = 0.15(0.6P - L) = 0.15(480,000 - L)$ if positive.
This can be written $0.15[480,000 - (L \wedge 480,000)]$. Then,
$E(\text{Bonus}) = 0.15[480,000 - E(L \wedge 480,000)]$
From Appendix A.2.3.1
E(Bonus)= 0.15{480,000 – [500,000 (1 – (500,000 / (480,000+500,000))]} = 35,265

Question # 97
Key: D

Severity after increase	Severity after increase and deductible
60	0
120	20
180	80
300	200

Expected payment per loss = 0.25(0) +0.25(20) + 0.25(80) + 0.25(200) = 75

Expected payments = Expected number of losses x Expected payment per loss
= 300(75) = 22,500

Question # 98
Key: A

E(S) = E(N)E(X) = 50(200) = 10,000

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 50(400) + 200^2(100) = 4{,}020{,}000$

$$\Pr(S < 8{,}000) \approx \Pr\left(Z < \frac{8{,}000 - 10{,}000}{\sqrt{4{,}020{,}000}} = -0.998 \right) = 0.16$$

Question #99
Key: B

Let S denote aggregate loss before deductible.

E(S) = 2(2) = 4, since mean severity is 2.

$f_S(0) = \dfrac{e^{-2}2^0}{0!} = 0.1353$, since must have 0 losses to get aggregate loss = 0.

$f_S(1) = \dfrac{e^{-2}2^1}{1!}\dfrac{1}{3} = 0.0902$, since must have 1 loss whose size is 1 to get aggregate loss = 1.

$$E(S \wedge 2) = 0 f_S(0) + 1 f_S(1) + 2[1 - f_S(0) - f_S(1)]$$
$$= 0(0.1353) + 1(0.0902) + 2(1 - 0.01353 - 0.0902) = 1.6392$$
$$E[(S-2)_+] = E(S) - E(S \wedge 2) = 4 - 1.6392 = 2.3608$$

Question #100
Key: C

Limited expected value =

$$\int_0^{1000} [1 - F(x)]dx = \int_0^{1000} 0.8e^{-0.02x} + 0.2e^{-0.001x} dx = -40e^{-0.02x} - 200e^{-0.001x} \Big|_0^{1000}$$
$$= -0 - 73.576 + 40 + 200 = 166.424$$

Question #101
Key: B

Mean excess loss = $\dfrac{E(X) - E(X \wedge 100)}{1 - F(100)} = \dfrac{331 - 91}{0.8} = 300$

Note that $E(X) = E(X \wedge 1000)$ because $F(1000) = 1$.

Question #102
Key: E

Expected insurance benefits per factory = $E[(X-1)_+] = 0.2(1) + 0.1(2) = 0.4$

Insurance premium = (1.1)(2 factories (0.4 per factory) = 0.88.

Let R = retained major repair costs, then

$$f_R(0) = 0.4^2 = 0.16,\ f_R(1) = 2(0.4)(0.6) = 0.48,\ f_S(2) = 0.6^2 = 0.36$$

Dividend = 3 – 0.88 – R – 0.15(3) = 1.67 – R, if positive.

Expected Dividend = 0.16(1.67 – 0) + (0.48)(1.67 – 1) + 0.36(0) = 0.5888

Question #103 - DELETED

Question: #104 - DELETED

Question: #105
Key: A

Using the conditional mean and variance formulas and that N has a conditional Poisson distribution:

$$0.2 = E(N) = E[E(N \mid \Lambda)] = E(\Lambda)$$
$$0.4 = Var(N) = E[Var(N \mid \Lambda)] + Var[E(N \mid \Lambda)] = E(\Lambda) + Var(\Lambda) = 0.2 + Var(\Lambda)$$
$$Var(\Lambda) = 0.4 - 0.2 = 0.2$$

Question: #106
Key: B

N = number of salmon in t hours
X = eggs from one salmon
S = total eggs.
$E(N) = 100t$
$Var(N) = 900t$

$$E(S) = E(N)\,\mathrm{E(X)} = (100\,\mathrm{t})(5) = 500\,\mathrm{t}$$

$$\mathrm{Var(S)} = \mathrm{E(N)\,Var(X)} + \mathrm{E(X)}^2 Var(N) = (100t)(5) + (5^2)(900t) = 23{,}000t$$

$$0.95 < \Pr(S > 10{,}000) = \Pr\left(Z > \frac{10{,}000 - 500t}{\sqrt{23{,}000t}}\right) \Rightarrow \frac{10{,}000 - 500t}{\sqrt{23{,}000t}} = -1.645$$

$$10{,}000 - 500t = -1.645(151.66)\sqrt{t} = -249.48\sqrt{t}$$

$$500t - 249.48\sqrt{t} - 10{,}000 = 0$$

$$\sqrt{t} = \frac{249.48 \pm \sqrt{(-249.48)^2 - 4(500)(-10{,}000)}}{2(500)} = 4.73$$

$$t = 22.26$$

Round up to 23

Question: #107
Key: C

X = losses on one life
$E(X) = 0.3(1) + 0.2(2) + 0.1(3) = 1$
S = total losses
$E(S) = 3E(X) = 3(1) = 3$

$$E[(S-1)_+] = E(S) - 1[1 - F_S(0)] = 3 - 1[1 - f_S(0)] = 3 - (1 - 0.4^3) = 2.064$$

Alternatively, the expected retained payment is $0 f_S(0) + 1[1 - f_S(0)] = 0.936$ and the stop-loss premium is 3 – 0.936 = 2.064.

Question: #108
Key: C

$$p(k) = \frac{2}{k}\,p(k-1) = \left(0 + \frac{2}{k}\right)p(k-1)$$

Thus an $(a, b, 0)$ distribution with $a = 0$, $b = 2$.

Thus Poisson with $\lambda = 2$.

$$p(4) = \frac{e^{-2}2^4}{4!} = 0.09$$

Question: #109
Key: B

By the memoryless property, the distribution of amounts paid in excess of 100 is still exponential with mean 200.

With the deductible, the probability that the amount paid is 0 is $F(100) = 1 - e^{-100/200} = 0.393$.

Thus the average amount paid per loss is (0.393)(0) + (0.607)(200) = 121.4
The expected number of losses is (20)(0.8) = 16.
The expected amount paid is (16)(121.4) = 1942.

Question: #110
Key: E

$$E(N) = 0.8(1) + 0.2(2) = 1.2$$
$$E(N^2) = 0.8(1) + 0.2(4) = 1.6$$
$$Var(N) = 1.6 - 1.2^2 = 0.16$$
$$E(X) = 0.2(0) + 0.7(100) + 0.1(1000) = 170$$
$$E(X^2) = 0.2(0) + 0.7(10,000) + 0.1(1,000,000) = 107,000$$
$$Var(X) = 107,000 - 170^2 = 78,100$$
$$E(S) = E(N)E(X) = 1.2(170) = 204$$
$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 1.2(78,100) + 170^2(0.16) = 98,344$$
$$SD(S) = \sqrt{98,344} = 313.6$$

So Budget = 204 + 314 = 518

Question: #111
Key: E

For a compound Poisson distribution, $Var(S) = E(N)E(X^2)$.

$E(X^2) = (1/2)(1) + (1/3)(4) + (1/6)(9) = 10/3$

$Var(S) = 12(10/3) = 40$

Alternatively, the total is the sum $N_1 + 2N_2 + 3N_3$ where the *N*s are independent Poisson variables with means 6, 4, and 2. The variance is 6 + 4(4) + 9(2) = 40.

Question: #112
Key: A

N = number of physicians	*E*(*N*) = 3	Var (*N*) = 2
X = visits per physician	E(*X*) = 30	Var (*X*) = 30
S = total visits		

$E(S) = E(N)E(X) = 3(30) = 90$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 3(30) + 900(2) = 1890$

$SD(S) = \sqrt{1890} = 43.5$

$\Pr(S > 119.5) = \Pr\left(Z > \frac{119.5 - 90}{43.5} = 0.68\right) = 1 - \Phi(0.68)$

Question: #113
Key: E

$E(N) = 0.7(0) + 0.2(2) + 0.1(3) = 0.7$

$Var(N) = 0.7(0) + 0.2(4) + 0.1(9) - 0.7^2 = 1.21$

$E(X) = 0.8(0) + 0.2(10) = 2$

$Var(X) = 0.8(0) + 0.2(100) - 2^2 = 16$

$E(S) = E(N)E(X) = 0.7(2) = 1.4$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 0.7(16) + 4(1.21) = 16.04$

$SD(S) = \sqrt{16.04} = 4$

$\Pr(S > 1.4 + 2(4) = 9.4) = 1 - \Pr(S = 0) = 1 - 0.7 - 0.2(0.8)^2 - 0.1(0.8)^3 = 0.12$

The last line follows because there are no possible values for *S* between 0 and 10. A value of 0 can be obtained three ways: no claims, two claims both for 0, three claims all for 0.

Question: #114
Key: A

$$P(0)=\frac{1}{5}\int_0^5 e^{-\lambda}d\lambda=\frac{1}{5}(-e^{-\lambda})\Big|_0^5=\frac{1}{5}(1-e^{-5})=0.1987$$
$$P(1)=\frac{1}{5}\int_0^5 \lambda e^{-\lambda}d\lambda=\frac{1}{5}(-\lambda e^{-\lambda}-\mathrm{e}^{-\lambda})\Big|_0^5=\frac{1}{5}(1-6e^{-5})=0.1919$$
$$P(N\geq 2)=1-0.1987-0.1919=0.6094$$

Question: #115
Key: D

Let X be the occurrence amount and Y = max(X – 100, 0) be the amount paid.
$E(X)=1000, Var(X)=1000^2$, $\Pr(X>100)=\mathrm{e}^{-100/1000}=0.904837$
The distribution of Y given that X > 100, is also exponential with mean 1,000 (memoryless property). So Y is 0 with probability 1 – 0.904837 = 0.095163 and is exponential with mean 1000 with probability 0.904837.
$$E(Y)=0.095163(0)+0.904837(1000)=904.837$$
$$E(Y^2)=0.095163(0)+0.904837(2)(1000^2)=1{,}809{,}675$$
$$Var(Y)=1{,}809{,}675-904.837^2=990{,}944$$

Question: #116
Key: C

Expected claims under current distribution = 500/(2 – 1) = 500
K= parameter of new distribution
X = claims
$$E(X)=K/(2-1)=K$$
$$E(\text{claims}+\text{bonus})=E\{X+0.5[500-(X\wedge 500)]\}=K+0.5\left[500-K\left(1-\frac{K}{500+K}\right)\right]=500$$
$$(K+250-0.5K)(500+K)+0.5K^2=500(500+K)$$
$$0.5K^2+500K+125{,}000+0.5K^2=250{,}000+500K$$
$$K^2=125{,}000$$
$$K=\sqrt{125{,}000}=354$$

Question: #117 - DELETED

Question: #118
Key: D

$$E(N)=25$$
$$Var(N)=25$$
$$E(X)=(5+95)/2=50$$
$$Var(X)=(95-5)^2/12=675$$
$$E(S)=E(N)E(X)=25(50)=1250$$
$$Var(S)=E(N)Var(X)+E(X)^2Var(N)=25(675)+50^2(25)=79,375$$
$$SD(S)=\sqrt{79,375}=281.74$$
$$P(S>2000)=P\left(Z>\frac{2000-1250}{281.74}=2.66\right)=1-\Phi(2.66)$$

Question: #119 - DELETED

Question: #120
Key: E

For 2001:
$$E(X)=2000/(2-1)=2000$$
$$E(X\wedge 3000)=\frac{2000}{1}\left(1-\frac{2000}{3000+2000}\right)=1200$$

So the fraction of the losses expected to be covered by the reinsurance is (2000 – 1200)/2000 = 0.4. The expected ceded losses are 4,000,000 and the ceded premium is 4,400,000.

For 2002:
Inflation changes the scale parameter, here to 2000(1.05) = 2100. The revised calculations are
$$E(X)=2100/(2-1)=2100$$
$$E(X\wedge 3000)=\frac{2100}{1}\left(1-\frac{2100}{3000+2100}\right)=1235$$
The revised premium is 1.1(10,500,000)(2100 – 1235)/2100 = 4,757,500.
The ratio is 4,757,500/4,400,000 = 1.08.

Question #121 - DELETED

Question #122 - DELETED

Question #123
Key: C

$$E\left(X\wedge x\right)=\frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right]=\frac{2000}{1}\left[1-\frac{2000}{x+2000}\right]=\frac{2000x}{x+2000}$$

x	$E(X\wedge x)$
∞	2000
250	222
2250	1059
5100	1437

$$0.75[E(X\wedge 2250)-E(X\wedge 250)]+0.95\left[E(X)-E(X\wedge 5100)\right]$$
$$0.75(1059-222)+0.95(2000-1437)=1162.6$$

The 5100 breakpoint was determined by when the insured’s share reaches 3600:

3600 = 250 + 0.25 (2250 – 250) + (5100 – 2250)

Question #124 - DELETED

Question #125
Key: A

N_I, N_{II} denote the random variables for # of claims for Types I and II in one year
X_I, X_{II} denote the claim amount random variables for Types I and II
S_I, S_{II} denote the total claim amount random variables for Types I and II
$S=S_I+S_{II}$
$E(N_I)=Var(N_I)=12,\ E(N_{II})=Var(N_{II})=4$
$E(X_I)=(0+1)/2=1/2, Var(X_I)=(1-0)^2/12=1/12$
$E(X_{II})=(0+5)/2=5/2, Var(X_{II})=(5-0)^2/12=25/12$
$E(S)=E(N_I)E(X_I)+E(N_{II})E(X_{II})=12(1/2)+4(5/2)=16$

$$Var(S)=E(N_I)Var(X_I)+E(X_I)^2Var(N_I)+E(N_{II})Var(X_{II})+E(X_{II})^2Var(N_{II})$$
$$=12(1/12)+(1/2)^2(12)+4(25/12)+(5/2)^2(4)=37.33$$

$$\Pr(S>18)=\Pr\left(Z>\frac{18-16}{\sqrt{37.33}}=0.327\right)=1-\Phi(0.327)=0.37$$

Question #126
Key: C

Let X be the loss random variable. Then, $(X-5)_+$ is the claim random variable.

$$E(X)=\frac{10}{2.5-1}=6.667$$

$$E(X\wedge 5)=\left(\frac{10}{2.5-1}\right)\left[1-\left(\frac{10}{5+10}\right)^{2.5-1}\right]=3.038$$

$$E[(X-5)_+]=E(X)-E(X\wedge 5)=6.667-3.038=3.629$$

Expected aggregate claims = $E(N)E[(X-5)_+]=5(3.629)=18.15.$

Question #127
Key: B

A Pareto $(\alpha=2,\theta=5)$ distribution with 20% inflation becomes Pareto with $\alpha=2,\theta=5(1.2)=6$. In 2004

$$E(X)=\frac{6}{2-1}=6$$

$$E(X\wedge 10)=\frac{6}{2-1}\left[1-\left(\frac{6}{10+6}\right)^{2-1}\right]=3.75$$

$$E[(X-10)_+]_+=E(X)-E(X\wedge 10)=6-3.75-2.25$$

$$\text{LER}=1-\frac{E[(X-10)_+]}{E(X)}=1-\frac{2.25}{6}=0.625$$

Question #128 - DELETED

Question #129 - DELETED

Question #130
Key: E

Begin with $E(W)=E(2^N)=E[E(2^N \mid \Lambda)]$. The inner expectation is the probability generating function of Poisson distribution evaluated at 2, $P_N(2)=e^{\Lambda(2-1)}=e^{\Lambda}$. Then,
$E(e^{\Lambda})=\int_0^4 e^{\lambda}(0.25)d\lambda = 0.25e^{\lambda}\Big|_0^4 = 0.25(e^4-1)=13.4$.

If the pgf is not recognized, the inner expectation can be derived as
$E(2^N \mid \Lambda)=\sum_{n=0}^{\infty} 2^n \frac{e^{-\Lambda}\Lambda^n}{n!} = \sum_{n=0}^{\infty}\frac{e^{-\Lambda}(2\Lambda)^n}{n!} = e^{\Lambda}\sum_{n=0}^{\infty}\frac{e^{-2\Lambda}(2\Lambda)^n}{n!} = e^{\Lambda}$ noting that the sum is over the probabilities for a Poisson distribution with mean 2Λ and so must be 1.

Question #131 - DELETED

Question #132 - DELETED

Question #133
Key: C

$$E(X \mid q)=3q, Var(X \mid q)=3q(1-q)$$
$$\mu = E(3q)=\int_0^1 3q2qdq = 2q^3\Big|_0^1 = 2$$
$$v = E[3q(1-q)] = \int_0^1 3q(1-q)2qdq = 2q^3 - 1.5q^4\Big|_0^1 = 0.5$$
$$a = Var(3q) = E(9q^2)-\mu^2 = \int_0^1 9q^2 2qdq - 2^2 = 4.5q^4\Big|_0^1 - 4 = 4.5-4 = 0.5$$
$$k = v/a = 0.5/0.5 = 1, Z = \frac{1}{1+1} = 0.5$$
The estimate is 0.5(0) + 0.5(2) = 1.

Question #134 - DELETED

Question #135 - DELETED

Question #136
Key: B

$$\Pr(\text{class 1} \mid \text{claim} = 250) = \frac{\Pr(\text{claim} = 250 \mid \text{class 1})\Pr(\text{class 1})}{\Pr(\text{claim} = 250 \mid \text{class 1})\Pr(\text{class 1}) + \Pr(\text{claim} = 250 \mid \text{class 2})\Pr(\text{class 2})}$$

$$= \frac{0.5(2/3)}{0.5(2/3) + 0.7(1/3)} = \frac{10}{17}$$

$$E(\text{claim} \mid \text{class 1}) = 0.5(250) + 0.3(2,500) + 0.2(60,000) = 12,875$$

$$E(\text{claim} \mid \text{class 2}) = 0.7(250) + 0.2(2,500) + 0.1(60,000) = 6,675$$

$$E(\text{claim} \mid 250) = (10/17)(12,875) + (7/17)(6,675) = 10,322$$

Question #137
Key: D

$$L(p) = f(0.74)f(0.81)f(0.95) = (p+1)0.74^p(p+1)0.81^p(p+1)0.95^p$$

$$= (p+1)^3(0.56943)^p$$

$$l(p) = \ln L(p) = 3\ln(p+1) + p\ln(0.56943)$$

$$l'(p) = \frac{3}{p+1} - 0.563119 = 0$$

$$p+1 = \frac{3}{0.563119} = 5.32747,\ p = 4.32747.$$

Question #138 - DELETED

Question #139
Key: C

Let X be the number of claims.

$E(X \mid I) = 0.9(0) + 0.1(2) = 0.2$

$E(X \mid II) = 0.8(0) + 0.1(1) + 0.1(2) = 0.3$

$E(X \mid III) = 0.7(0) + 0.2(1) + 0.1(2) = 0.4$

$Var(X \mid I) = 0.9(0) + 0.1(4) - 0.2^2 = 0.36$

$Var(X \mid II) = 0.8(0) + 0.1(1) + 0.1(4) - 0.3^2 = 0.41$

$Var(X \mid III) = 0.7(0) + 0.2(1) + 0.1(4) - 0.4^2 = 0.44.$

$\mu = (1/2)(0.2 + 0.3 + 0.4) = 0.3$

$v = (1/3)(0.36 + 0.41 + 0.44) = 0.403333$

$a = (1/3)(0.2^2 + 0.3^2 + 0.4^2) - 0.3^2 = 0.006667$

$k = 0.403333 / 0.006667 = 60.5$

$$Z = \frac{50}{50 + 60.5} = 0.45249.$$

For one insured the estimate is 0.45249(17/50) + 0.54751(0.3) = 0.31810.
For 35 insureds the estimate is 35(0.31810) = 11.13.

Question #140
Key: A

For the given intervals, based on the model probabilities, the expected counts are 4.8, 3.3, 8.4, 7.8, 2.7, 1.5, and 1.5. To get the totals above 5, group the first two intervals and the last three. The table is

Interval	Observed	Expected	Chi-square
0-500	3	8.1	3.21
500-2498	8	8.4	0.02
2498-4876	9	7.8	0.18
4876-infinity	10	5.7	3.24
Total	30	30	6.65

Question #141 - DELETED

Question #142
Key: C

$$0.575 = \Pr(N=0) = \int_0^k \Pr(N=0\,|\,\theta)\pi(\theta)d\theta$$

$$= \int_0^k e^{-\theta}\frac{e^{-\theta}}{1-e^{-k}}d\theta = -\frac{e^{-2\theta}}{2(1-e^{-k})}\Bigg|_0^k = -\frac{e^{-2k}}{2(1-e^{-k})} + \frac{1}{2(1-e^{-k})}$$

$$= \frac{1-e^{-2k}}{2(1-e^{-k})} = \frac{1+e^{-k}}{2}$$

$$e^{-k} = 2(0.575) - 1 = 0.15$$

$$k = 1.90.$$

Question #143 - DELETED

Question #144 - DELETED

Question #145
Key: B

The subscripts denote the three companies.

$$x_{I1} = \frac{50,000}{100} = 500, \quad x_{I2} = \frac{50,000}{200} = 250, \quad x_{II1} = \frac{150,000}{500} = 300$$

$$x_{II2} = \frac{150,000}{300} = 500, \quad x_{III1} = \frac{150,000}{50} = 3,000, \quad x_{III2} = \frac{150,000}{150} = 1,000$$

$$\bar{x}_I = \frac{100,000}{300} = 333.33, \quad \bar{x}_{II} = \frac{300,000}{800} = 375, \quad \bar{x}_{III} = \frac{300,000}{200} = 1,500, \quad \bar{x} = \frac{700,000}{1,300} = 538.46$$

$$\hat{v} = \frac{\begin{array}{c}100(500-333.33)^2 + 200(250-333.33)^2 + 500(300-375)^2 + 300(500-375)^2 \\ +50(3,000-1,500)^2 + 150(1,000-1,500)^2\end{array}}{(2-1)+(2-1)+(2-1)} = 53,888,888.89$$

$$\hat{a} = \frac{\begin{array}{c}300(333.33-538.46)^2 + 800(375-538.46)^2 + 200(1,500-538.46)^2 \\ -53,888,888.89(3-1)\end{array}}{1,300 - \dfrac{300^2+800^2+200^2}{1,300}} = 157,035.60$$

$$k = \frac{53,888,888.89}{157,035.60} = 343.1635, \quad Z = \frac{200}{200+343.1635} = 0.3682$$

Question #146
Key: D

Let α_j be the parameter for region *j*. The likelihood function is $L = \left(\prod_{i=1}^{n} \frac{\alpha_1}{x_i^{\alpha_1+1}}\right)\left(\prod_{i=1}^{m} \frac{\alpha_2}{y_i^{\alpha_2+1}}\right)$.

The expected values satisfy $\frac{\alpha_2}{\alpha_2 - 1} = 1.5\frac{\alpha_1}{\alpha_1 - 1}$ and so $\alpha_2 = \frac{3\alpha_1}{2+\alpha_1}$. Substituting this in the likelihood function and taking logs produces

$$l(\alpha_1) = \ln L(\alpha_1) = n\ln\alpha_1 - (\alpha_1+1)\sum_{i=1}^{n}\ln x_i + m\ln\left(\frac{3\alpha_1}{2+\alpha_1}\right) - \frac{2+4\alpha_1}{2+\alpha_1}\sum_{i=1}^{m}\ln y_i$$

$$l'(\alpha_1) = \frac{n}{\alpha_1} - \sum_{i=1}^{n}\ln x_i + \frac{2m}{\alpha_1(2+\alpha_1)} - \frac{6}{(2+\alpha_1)^2}\sum_{i=1}^{m}\ln y_i = 0.$$

Question #147 - DELETED

Question #148
Key: B

The mean is mq and the variance is $mq(1 - q)$. The mean is 34,574 and so the full credibility standard requires the confidence interval to be ±345.74 which must be 1.96 standard deviations. Thus,

$$345.74 = 1.96\sqrt{mq(1-q)} = 1.96\sqrt{34,574}\sqrt{1-q}$$

$$1-q = 0.9, \quad q = 0.1.$$

Question #149 - DELETED

Question #150 - DELETED

Question #151
Key: C

$$E(N|1) = 5,\ E(N|2) = 8(0.55) = 4.4,\ \mu = 0.5(5) + 0.5(4.4) = 4.7$$

$$Var(N|1) = 5, Var(N|2) = 8(0.55)(0.45) = 1.98,\ v = 0.5(5) + 0.5(1.98) = 3.49$$

$$a = 0.5(5)^2 + 0.5(4.4)^2 - 4.7^2 = 0.09,\ k = 3.49/0.09 = 38.7778$$

$$Z = \frac{3}{3+38.7778} = 0.0718,\ 4.6019 = 0.0718\frac{7+r}{3} + 0.9282(4.7)$$

$$Var(N|1) = 5, Var(N|2) = 8(0.55)(0.45) = 1.98,\ v = 0.5(5) + 0.5(1.98) = 3.49$$

$$a = 0.5(5)^2 + 0.5(4.4)^2 - 4.7^2 = 0.09,\ k = 3.49/0.09 = 38.7778$$

$$Z = \frac{3}{3+38.7778} = 0.0718,\ 4.6019 = 0.0718\frac{7+r}{3} + 0.9282(4.7)$$

The solution is $r = 3$.

Question #152

Key: A

These observations are truncated at 500. The contribution to the likelihood function is $\frac{f(x)}{1-F(500)} = \frac{\theta^{-1}e^{-x/\theta}}{e^{-500/\theta}}$. Then the likelihood function is

$$L(\theta) = \frac{\theta^{-1}e^{-600/\theta}\theta^{-1}e^{-700/\theta}\theta^{-1}e^{-900/\theta}}{\left(e^{-500/\theta}\right)^3} = \theta^{-3}e^{-700/\theta}$$

$$l(\theta) = \ln L(\theta) = -3\ln\theta - 700\theta^{-1}$$

$$l'(\theta) = -3\theta^{-1} + 700\theta^{-2} = 0$$

$$\theta = 700/3 = 233.33.$$

Question #153 - DELETED

Question #154
Key: E

For a compound Poisson distribution, S, the mean is $E(S\,|\,\lambda,\mu,\sigma) = \lambda E(X) = \lambda e^{\mu+0.5\sigma^2}$ and the variance is $Var(S\,|\,\lambda,\mu,\sigma) = \lambda E(X^2) = \lambda e^{2\mu+2\sigma^2}$. Then,

$$E(S) = E[E(S\,|\,\lambda,\mu,\sigma)] = \int_0^1\int_0^1\int_0^1 \lambda e^{\mu+0.5\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma$$

$$= \int_0^1\int_0^1 e^{\mu+0.5\sigma^2}\sigma\, d\mu\, d\sigma = \int_0^1 (e-1)e^{0.5\sigma^2}\sigma\, d\sigma$$

$$= (e-1)(e^{0.5}-1) = 1.114686$$

$$v = E[Var(S\,|\,\lambda,\mu,\sigma)] = \int_0^1\int_0^1\int_0^1 \lambda e^{2\mu+2\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma$$

$$= \int_0^1\int_0^1 e^{2\mu+2\sigma^2}\sigma\, d\mu\, d\sigma = \int_0^1 0.5(e^2-1)e^{2\sigma^2}\sigma\, d\sigma$$

$$= 0.5(e^2-1)0.25(e^2-1) = 0.125(e^2-1)^2 = 5.1025$$

$$a = Var[E(S\,|\,\lambda,\mu,\sigma)] = \int_0^1\int_0^1\int_0^1 \lambda^2 e^{2\mu+\sigma^2}\,2\sigma\, d\lambda\, d\mu\, d\sigma - E(S)^2$$

$$= \int_0^1\int_0^1 \frac{2}{3}e^{2\mu+\sigma^2}\sigma\, d\mu\, d\sigma - E(S)^2 = \int_0^1 \frac{1}{3}(e^2-1)e^{\sigma^2}\sigma\, d\sigma - E(S)^2$$

$$= \frac{1}{3}(e^2-1)\frac{1}{2}(e-1) - E(S)^2 = (e^2-1)(e-1)/6 - E(S)^2 = 0.587175$$

$$k = \frac{5.1025}{0.587175} = 8.69.$$

Question #155 - DELETED

Question #156
Key: C

There are $n/2$ observations of $N = 0$ (given $N = 0$ or 1) and $n/2$ observations of $N = 1$ (given $N = 0$ or 1). The likelihood function is

$L = \left(\frac{e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2}\left(\frac{\lambda e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}}\right)^{n/2} = \frac{\lambda^{n/2}e^{-n\lambda}}{(e^{-\lambda}+\lambda e^{-\lambda})^n} = \frac{\lambda^{n/2}}{(1+\lambda)^n}$. Taking logs, differentiating and solving provides the answer.

$$l = \ln L = (n/2)\ln\lambda - n\ln(1+\lambda)$$

$$l' = \frac{n}{2\lambda} - \frac{n}{1+\lambda} = 0$$

$$n(1+\lambda) - n2\lambda = 0$$

$$1-\lambda = 0, \quad \lambda = 1.$$

Question #157
Key: D

The posterior density function is proportional to the product of the likelihood function and prior density. That is, $\pi(q\,|\,1,0) \propto f(1\,|\,q)f(0\,|\,q)\pi(q) \propto q(1-q)q^3 = q^4 - q^5$. To get the exact posterior density, integrate this function over its range:

$\int_{0.6}^{0.8} q^4 - q^5\,dq = \left.\frac{q^5}{5} - \frac{q^6}{6}\right|_{0.6}^{0.8} = 0.014069$ and so $\pi(q\,|\,1,0) = \frac{q^4 - q^5}{0.014069}$. Then,

$$\Pr(0.7 < q < 0.8\,|\,1,0) = \int_{0.7}^{0.8} \frac{q^4 - q^5}{0.014069}\,dq = 0.5572.$$

Question #158 - DELETED

Question #159
Key: A

The sample mean is $\frac{0(2000)+1(600)+2(300)+3(80)+4(20)}{3000} = 0.5066667 = \hat{\mu} = \hat{v}$ and the sample variance is

$\frac{2000(0-\hat{\mu})^2 + 600(1-\hat{\mu})^2 + 300(2-\hat{\mu})^2 + 80(3-\hat{\mu})^2 + 20(4-\hat{\mu})^2}{2999} = 0.6901856.$ Then,

$\hat{a} = 0.6901856 - 0.5066667 = 0.1835189,\ k = \frac{0.5066667}{0.1835189} = 2.760842$ and

$$Z = \frac{1}{1+2.760842} = 0.2659.$$

Question #160

Key: E

The cdf is $F(x)=\int_0^x 4(1+t)^{-5}dt=-(1+t)^{-4}\Big|_0^x=1-\frac{1}{(1+x)^4}$.

Observation (x)	$F(x)$	compare to:	Maximum difference
0.1	0.317	0, 0.2	0.317
0.2	0.518	0.2, 0.4	0.318
0.5	0.802	0.4, 0.6	0.402
0.7	0.880	0.6, 0.8	0.280
1.3	0.964	0.8, 1.0	0.164

K-S statistic is 0.402.

Question #161 - DELETED

Question #162
Key: B

$E[X-d\mid X>d]$ is the expected payment per payment with an ordinary deductible of d
It can be evaluated (for Pareto) as

$$\frac{E(X)-E(X\wedge d)}{1-F(d)}=\frac{\frac{\theta}{\alpha-1}-\frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}\right]}{1-\left[1-\left(\frac{\theta}{d+\theta}\right)^{\alpha}\right]}=\frac{\frac{\theta}{\alpha-1}\left(\frac{\theta}{d+\theta}\right)^{\alpha-1}}{\left(\frac{\theta}{d+\theta}\right)^{\alpha}}=\frac{d+\theta}{\alpha-1}=d+\theta$$

$E[X-100\mid X>100]=(5/3)E[X-50\mid X>50]$

$100+\theta=(5/3)(50+\theta)$

$300+3\theta=250+5\theta, 50=2\theta, \theta=25$

$E[X-150\mid X>150]=150+\theta=150+25=175$

Question #163
Key: D

Let S = score

$E(S) = E[E(S \mid \theta)] = E(\theta) = 75$

$Var(S) = E[Var(S \mid \theta)] + Var[E(S \mid \theta)] = E(8^2) + Var(\theta) = 64 + 6^2 = 100$

S is normally distributed (a normal mixture of normal distributions with constant variance is normal).

$$\Pr(S < 90 \mid S > 65) = \frac{F(90) - F(65)}{1 - F(65)} = \frac{\Phi[(90-75)/10)] - \Phi[(65-75)/10]}{1 - \Phi[(65-75)/10]}$$

$$\frac{\Phi(1.5) - \Phi(-1.0)}{1 - \Phi(-1.0)} = \frac{0.9332 - (1 - 0.8413)}{1 - (1 - 0.8413)} = 0.9206$$

Question #164
Key: B

Losses in excess of the deductible occur at a Poisson rate of $\lambda^* = [1 - F(30)]\lambda = 0.75(20) = 15$
The expected payment squared per payment is

$$E[(X-30)^2 \mid X > 30) = E(X^2 - 60X + 900 \mid X > 30)$$

$$E[X^2 - 60(X-30) - 900 \mid X > 30]$$

$$E[X^2 \mid X > 30] - 60\frac{E(X) - E(X \wedge 30)}{1 - F(30)} - 900$$

$$= 9000 - 60\frac{70-25}{0.75} - 900 = 4500$$

The variance of S is the expected number of payments times the second moment, 15(4500) = 67,500.

Question #165
Key: A

$$E[(S-3)_+] = E(S) - 3 + 3f_S(0) + 2f_S(1) + 1f_S(2)$$

$$E(S) = 2[0.6 + 2(0.4)] = 2.8$$

$$f_S(0) = e^{-2},\ f_S(1) = e^{-2}(2)(0.6) = 1.2e^{-2}$$

$$f_S(2) = e^{-2}(2)(0.4) + \frac{e^{-2}2^2}{2!}(0.6)^2 = 1.52e^{-2}$$

$$E[(S-3)_+] = 2.8 - 3 + 3e^{-2} + 2(1.2e^{-2}) + 1(1.52e^{-2}) = -0.2 + 6.92e^{-2} = 0.7365$$

Question #166
Key: C

Write (i) as $\frac{p_k}{p_{k-1}} = c + \frac{c}{k}$

This is an $(a, b, 0)$ distribution with $a = b = c$.

Which?

1. If Poisson, $a = 0$, so $b = 0$ and thus $p_0 = 0.5$ and $p_1 = p_2 = \cdots = 0$. The probabilities do not sum to 1 and so not Poisson.
2. If Geometric, $b = 0$, so $a = 0$. By same reasoning as #1, not Geometric.
3. If binomial, a and b have opposite signs. But here $a = b$, so not binomial.
4. Thus negative binomial.

   $1 = \frac{a}{b} = \frac{\beta/(1+\beta)}{(r-1)\beta/(1-\beta)} = \frac{1}{r-1}$ so $r = 2$.

   $p_0 = 0.5 = (1+\beta)^{-r} = (1+\beta)^{-2} \Rightarrow \beta = \sqrt{2} - 1 = 0.414$

   $c = a = \beta/(1+\beta) = 0.29$

Question #167
Key: B

The number of repairs for each boat type has a binomial distribution.

For power boats:

$E(S) = 100(0.3)(300) = 9,000,$

$Var(S) = 100(0.3)10,000) + 100(0.3)(0.7)(300^2) = 2,190,000$

For sail boats:

$E(S) = 300(0.1)(1,000) = 30,000,$

$Var(S) = 300(0.1)(400,000) + 300(0.1)(0.9)(1,000^2) = 39,000,000$

For luxury yachts:

$E(S) = 50(0.6)(5,000) = 150,000,$

$Var(S) = 50(0.6)(0.4)(2,000,000) + 50(0.6)(0.4)(5,000^2) = 360,000,000$

The sums are 189,000 expected and a variance of 401,190,000 for a standard deviation of 20,030. The mean plus standard deviation is 209,030.

Question #168
Key: B

$$S_X(150) = 1 - 0.2 = 0.8$$

$$f_{Y^P}(y) = \frac{f_X(y+150)}{S_X(150)},\ f_{Y^P}(50) = \frac{0.2}{0.8} = 0.25,\ f_{Y^P}(150) = \frac{0.6}{0.8} = 0.75$$

$$E(Y^P) = 0.25(50) + 0.75(150) = 125$$

$$E\left[(Y^P)^2\right] = 0.25(50^2) + 0.75(150^2) = 17{,}500$$

$$Var(Y^P) = 17{,}500 - 125^2 = 1{,}875$$

Question #169
Key: A

$$F(200) = 0.8\left[1 - \left(\frac{100}{200+100}\right)^2\right] + 0.2\left[1 - \left(\frac{3000}{3000+200}\right)^4\right] = 0.7566$$

Question #170
Key: B

Let *C* denote child; *ANS* denote Adult Non-Smoker; *AS* denote Adult Smoker.

$$P(3\,|\,C)P(C) = \frac{3^3 e^{-3}}{3!}(0.3) = 0.067$$

$$P(3\,|\,ANS)P(ANS) = \frac{1^3 e^{-1}}{3!}(0.6) = 0.037$$

$$P(3\,|\,AS)P(AS) = \frac{4^3 e^{-4}}{3!}(0.1) = 0.020$$

$$P(AS\,|\,N=3) = \frac{0.020}{0.067 + 0.037 + 0.020} = 0.16$$

Question #171
Key: C

$$E(S) = E(N)E(X) = 3(10) = 30$$

$$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 3\frac{400}{12} + 100(3.6) = 100 + 360 = 460$$

For 95th percentile, $E(S) + 1.645\sqrt{Var(S)} = 30 + 1.645\sqrt{460} = 65.28$

Question #172
Key: D

The CDF is $F(x) = 1 - \frac{1}{(1+x)^4}$

Observation (x)	$F(x)$	compare to:	Maximum difference
0.2	0.518	0, 0.2	0.518
0.7	0.880	0.2, 0.4	0.680
0.9	0.923	0.4, 0.6	0.523
1.1	0.949	0.6, 0.8	0.349
1.3	0.964	0.8, 1.0	0.164

The K-S statistic is the maximum from the last column, 0.680.
Critical values are: 0.546, 0.608, 0.662, and 0.729 for the given levels of significance. The test statistic is between 0.662 (2.5%) and 0.729 (1.0%) and therefore the test is rejected at 0.025 and not at 0.01.

Question #173
Key: E

For claim severity,
$\mu_S = 1(0.4) + 10(0.4) + 100(0.2) = 24.4,$
$\sigma_S^2 = 1^2(0.4) + 10^2(0.4) + 100^2(0.2) - 24.4^2 = 1,445.04.$
For claim frequency,
$\mu_F = r\beta = 3r, \quad \sigma_F^2 = r\beta(1+\beta) = 12r,$
For aggregate losses,
$\mu = \mu_S \mu_F = 24.4(3r) = 73.2r,$
$\sigma^2 = \mu_S^2 \sigma_F^2 + \sigma_S^2 \mu_F = 24.4^2(12r) + 1,445.04(3r) = 11,479.44r.$
For the given probability and tolerance, $\lambda_0 = (1.96 / 0.1)^2 = 384.16.$
The number of observations needed is
$\lambda_0 \sigma^2 / \mu^2 = 384.16(11,479.44r) / (73.2r)^2 = 823.02 / r.$
The average observation produces $3r$ claims and so the required number of claims is $(823.02 / r)(3r) = 2,469.$

Question #174 - DELETED

Question #175 - DELETED

Question #176
Key: A

Pick one of the points, say the fifth one. The vertical coordinate is $F(30)$ from the model and should be slightly less than 0.6. Inserting 30 into the five answers produces 0.573, 0.096, 0.293, 0.950, and something less than 0.5. Only the model in answer A is close.

Question #177
Key: E

The distribution of Θ is Pareto with parameters 1 and 2.6. Then,

$$v = EPV = E(\Theta) = \frac{1}{2.6-1} = 0.625, \quad a = VHM = Var(\Theta) = \frac{2}{1.6(0.6)} - 0.625^2 = 1.6927,$$

$$k = v/a = 0.625/1.6927 = 0.3692, \quad Z = \frac{5}{5+0.3692} = 0.9312.$$

Question #178 - DELETED

Question #179
Key: D

For an exponential distribution, the maximum likelihood estimate of θ is the sample mean, 6. Let $Y = X_1 + X_2$ where each X has an exponential distribution with mean 6. The sample mean is $Y/2$ and Y has a gamma distribution with parameters 2 and 6. Then

$$\Pr(Y/2 > 10) = \Pr(Y > 20) = \int_{20}^{\infty} \frac{xe^{-x/6}}{36}dx$$

$$= -\frac{xe^{-x/6}}{6} - e^{-x/6}\Big|_{20}^{\infty} = \frac{20e^{-20/6}}{6} + e^{-20/6} = 0.1546.$$

Question #180
Key: A

Let $W = X_1 + X_2$ where each *X* has an exponential distribution with mean θ. The sample mean is *Y* = *W*/2 and *W* has a gamma distribution with parameters 2 and θ. Then

$$g(\theta) = F_Y(10) = \Pr(Y \le 10) = \Pr(W \le 20) = \int_0^{20} \frac{we^{-w/\theta}}{\theta^2}\,dw$$

$$= -\frac{we^{-w/\theta}}{\theta} - e^{-w/\theta}\Bigg|_0^{20} = 1 - \frac{20e^{-20/\theta}}{\theta} - e^{-20/\theta} = 1 - \mathrm{e}^{-20/\theta}(1 + 20\theta^{-1}).$$

$$g'(\theta) = -\frac{20}{\theta^2}e^{-20/\theta}(1 + 20\theta^{-1}) + e^{-20/\theta}\frac{20}{\theta^2} = -\frac{400e^{-20/\theta}}{\theta^3}.$$

At the maximum likelihood estimate of 6, $g'(6) = -0.066063.$

The maximum likelihood estimator is the sample mean. Its variance is the variance of one observation divided by the sample size. For the exponential distribution the variance is the square of the mean, so the estimated variance of the sample mean is 36/2 = 18. The answer is $(-0.066063)^2(18) = 0.079$.

Question #181
Key B

$$\mu(\lambda,\theta) = E(S \mid \lambda,\theta) = \lambda\theta,$$
$$v(\lambda,\theta) = Var(S \mid \lambda,\theta) = \lambda 2\theta^2,$$
$$v = EPV = E(\lambda 2\theta^2) = 1(2)(1+1) = 4,$$
$$a = VHM = Var(\lambda\theta) = E(\lambda^2)E(\theta^2) - [E(\lambda)E(\theta)]^2 = 2(2) - 1 = 3,$$
$$k = v/a = 4/3.$$

Question #182 - DELETED

Question #183 - DELETED

Question #184
Key: D

The posterior distribution can be found from

$$\pi(\lambda\,|\,10) \propto \frac{e^{-\lambda}\lambda^{10}}{10!}\left(\frac{0.4}{6}e^{-\lambda/6}+\frac{0.6}{12}e^{-\lambda/12}\right) \propto \lambda^{10}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right).$$

The required constant is

$$\int_0^\infty \lambda^{10}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right)d\lambda = 0.8(10!)(6/7)^{11}+0.6(10!)(12/13)^{11} = 0.395536(10!).$$

The posterior mean is

$$E(\lambda\,|\,10) = \frac{1}{0.395536(10!)}\int_0^\infty \lambda^{11}\left(0.8e^{-7\lambda/6}+0.6e^{-13\lambda/12}\right)d\lambda$$

$$= \frac{0.8(11!)(6/7)^{12}+0.6(11!)(12/13)^{12}}{0.395536(10!)} = 9.88.$$

Question #185 - DELETED

Question #186 - DELETED

Question #187
Key: D

For the geometric distribution $\mu(\beta)=\beta$ and $v(\beta)=\beta(1+\beta)$. The prior density is Pareto with parameters α and 1. Then,

$$\mu = E(\beta) = \frac{1}{\alpha-1},$$

$$v = EPV = E[\beta(1+\beta)] = \frac{1}{\alpha-1}+\frac{2}{(\alpha-1)(\alpha-2)} = \frac{\alpha}{(\alpha-1)(\alpha-2)},$$

$$a = VHM = Var(\beta) = \frac{2}{(\alpha-1)(\alpha-2)}-\frac{1}{(\alpha-1)^2} = \frac{\alpha}{(\alpha-1)^2(\alpha-2)},$$

$$k = v/a = \alpha-1, \quad Z = \frac{1}{1+k} = \frac{1}{\alpha}.$$

The estimate is

$$\frac{1}{\alpha}x+\left(1-\frac{1}{\alpha}\right)\frac{1}{\alpha-1} = \frac{x+1}{\alpha}.$$

Question #188 - DELETED

Question #189
Key: E

A is false. Using sample data gives a better than expected fit and therefore a test statistic that favors the null hypothesis, thus increasing the Type II error probability. The K-S test works only on individual data and so B is false. D is false because the critical value depends on the degrees of freedom which in turn depends on the number of cells, not the sample size.

Question #190
Key: B

$$E(\theta) = 0.05(0.8) + 0.3(0.2) = 0.1,$$
$$E(\theta^2) = 0.05^2(0.8) + 0.3^2(0.2) = 0.02,$$
$$\mu(\theta) = 0(2\theta) + 1(\theta) + 2(1-3\theta) = 2-5\theta,$$
$$v(\theta) = 0^2(2\theta) + 1^2(\theta) + 2^2(1-3\theta) - (2-5\theta)^2 = 9\theta - 25\theta^2,$$
$$\mu = E(2-5\theta) = 2-5(0.1) = 1.5,$$
$$v = EPV = E(9\theta - 25\theta^2) = 9(0.1) - 25(0.02) = 0.4,$$
$$a = VHM = Var(2-5\theta) = 25Var(\theta) = 25(0.02 - 0.1^2) = 0.25,$$
$$k = v/a = 0.4/0.25 = 1.6,\ Z = \frac{1}{1+1.6} = \frac{5}{13},$$
$$P = \frac{5}{13}2 + \frac{8}{13}1.5 = 1.6923.$$

Question #191
Key: B

$f(\lambda \mid 5,3) \propto \frac{e^{-\lambda}\lambda}{5!}\frac{e^{-\lambda}\lambda^3}{3!}\frac{2^5\lambda^5 e^{-2\lambda}}{24\lambda} \propto \lambda^{12}e^{-4\lambda}$. This is a gamma distribution with parameters 13 and 0.25. The expected value is 13(0.25) = 3.25.

Alternatively, if the Poisson-gamma relationships are known, begin with the prior parameters $\alpha = 5$ and $\beta = 2$ where $\beta = 1/\theta$ if the parameterization from *Loss Models* is considered. Then the posterior parameters are $\alpha' = 5+5+3 = 13$ where the second 5 and the 3 are the observations and $\beta' = 2+2 = 4$ where the second 2 is the number of observations. The posterior mean is then 13/4 = 3.25.

Question #192 - DELETED

Question #193 - DELETED

Question #194
Key: B

$$\hat{v} = \frac{50(200-227.27)^2 + 60(250-227.27)^2 + 100(160-178.95)^2 + 90(200-178.95)^2}{1+1}$$

$$= 71{,}985.647,$$

$$\hat{k} = 71{,}985.647 / 651.03 = 110.57,$$

$$\hat{Z} = \frac{110}{110+110.57} = 0.499.$$

Question #195 - DELETED

Question #196
Key: C

$$L = \left[\frac{f(750)}{1-F(200)}\right]^3 f(200)^3 f(300)^4 [1-F(10{,}000)]^6 \left[\frac{f(400)}{1-F(300)}\right]^4$$

$$= \left[\frac{\alpha 10{,}200^{\alpha}}{10{,}750^{\alpha+1}}\right]^3 \left[\frac{\alpha 10{,}000^{\alpha}}{10{,}200^{\alpha+1}}\right]^3 \left[\frac{\alpha 10{,}000^{\alpha}}{10{,}300^{\alpha+1}}\right]^4 \left[\frac{10{,}000^{\alpha}}{20{,}000^{\alpha}}\right]^6 \left[\frac{\alpha 10{,}300^{\alpha}}{10{,}400^{\alpha+1}}\right]^4$$

$$= \alpha^{14} 10{,}200^{-3} 10{,}000^{13\alpha} 10{,}300^{-4} 10{,}750^{-3\alpha-3} 20{,}000^{-6\alpha} 10{,}400^{-4\alpha-4}$$

$$\propto \alpha^{14} 10{,}000^{13\alpha} 10{,}750^{-3\alpha} 20{,}000^{-6\alpha} 10{,}400^{-4\alpha},$$

$$\ln L = 14 \ln \alpha + 13\alpha \ln(10{,}000) - 3\alpha \ln(10{,}750) - 6\alpha \ln(20{,}000) - 4\alpha \ln(10{,}400)$$

$$= 14 \ln \alpha - 4.5327\alpha.$$

The derivative is $14/\alpha - 4.5327$ and setting it equal to zero gives $\hat{\alpha} = 3.089$.

Question #197
Key: C

$$\hat{v} = \bar{x} = \frac{30+30+12+4}{100} = 0.76.$$

$$\hat{a} = \frac{50(0-0.76)^2 + 30(1-0.76)^2 + 15(2-0.76)^2 + 4(3-0.76)^2 + 1(4-0.76)^2}{99} - 0.76$$

$$= 0.090909,$$

$$\hat{k} = \frac{0.76}{0.090909} = 8.36, \quad \hat{Z} = \frac{1}{1+8.36} = 0.10684,$$

$$P = 0.10684(1) + 0.89316(0.76) = 0.78564.$$

The above analysis was based on the distribution of total claims for two years. Thus 0.78564 is the expected number of claims for the next two years. For the next one year the expected number is 0.78564/2 = 0.39282.

Question #198 - DELETED

Question #199
Key: E

The density function is $f(x)=\dfrac{0.2x^{-0.8}}{\theta^{0.2}}e^{-(x/\theta)^{0.2}}$. The likelihood function is

$$
\begin{aligned}
L(\theta) &= f(130)f(240)f(300)f(540)[1-F(1000)]^2 \\
&= \frac{0.2(130)^{-0.8}}{\theta^{0.2}}e^{-(130/\theta)^{0.2}}\frac{0.2(240)^{-0.8}}{\theta^{0.2}}e^{-(240/\theta)^{0.2}}\frac{0.2(300)^{-0.8}}{\theta^{0.2}}e^{-(300/\theta)^{0.2}}\frac{0.2(540)^{-0.8}}{\theta^{0.2}}e^{-(540/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}}e^{-(1000/\theta)^{0.2}} \\
&\propto \theta^{-0.8}e^{-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})},
\end{aligned}
$$

$$
\begin{aligned}
l(\theta) &= -0.8\ln(\theta)-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2}) \\
&= -0.8\ln(\theta)-20.2505\theta^{-0.2},
\end{aligned}
$$

$$l'(\theta)=-0.8\theta^{-1}+0.2(20.2505)\theta^{-1.2}=0,$$

$$\theta^{-0.2}=0.197526,\quad \hat{\theta}=3,325.67.$$

Question #200
Key: A

Buhlmann estimates are on a straight line, which eliminates E. Bayes estimates are never outside the range of the prior distribution. Because graphs B and D include values outside the range 1 to 4, they cannot be correct. The Buhlmann estimates are the linear least squares approximation to the Bayes estimates. In graph C the Bayes estimates are consistently higher and so the Buhlmann estimates are not the best approximation. This leaves A as the only feasible choice.

Question #201
Key: C

The expected counts are 300(0.035) = 10.5, 300(0.095) = 28.5, 300(0.5) = 150, 300(0.2) = 60, and 300(0.17) = 51 for the five groups. The test statistic is

$$\frac{(5-10.5)^2}{10.5}+\frac{(42-28.5)^2}{28.5}+\frac{(137-150)^2}{150}+\frac{(66-60)^2}{60}+\frac{(50-51)^2}{51}=11.02.$$

There are 5 – 1 = 4 degrees of freedom. From the table, the critical value for a 5% test is 9.488 and for a 2.5% test is 11.143. The hypothesis is rejected at 5%, but not at 2.5%.

Question #202 - DELETED

Question #203
Key: C

For the geometric distribution, $\Pr(X_1 = 2 \mid \beta) = \dfrac{\beta^2}{(1+\beta)^3}$ and the expected value is β.

$$\Pr(\beta = 2 \mid X_1 = 2) = \frac{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2)}{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2) + \Pr(X_1 = 2 \mid \beta = 5)\Pr(\beta = 5)}$$

$$= \frac{\frac{4}{27}\frac{1}{3}}{\frac{4}{27}\frac{1}{3} + \frac{25}{216}\frac{2}{3}} = 0.39024.$$

The expected value is then 0.39024(2) + 0.60976(5) = 3.83.

Question #204
Key: D

Let *X* be the random variable for when the statistic is forgotten. Then $F_X(x \mid y) = 1 - e^{-xy}$
For the unconditional distribution of *X*, integrate with respect to y

$$F_X(x) = \int_0^\infty (1 - e^{-xy}) \frac{1}{\Gamma(2)y}\left(\frac{y}{2}\right)^2 e^{-y/2} dy = 1 - \frac{1}{4}\int_0^\infty y e^{-y(x+1/2)} dy = 1 - \frac{1}{4(x+1/2)^2}$$

$$F(1/2) = 1 - \frac{1}{4(1/2 + 1/2)^2} = 0.75$$

There are various ways to evaluate the integral (1) Integration by parts. (2) Recognize that $\int_0^\infty y(x+1/2)e^{-y(x+1/2)}dy$ is the expected value of an exponential random variable with mean $(x+1/2)^{-1}$. (3) Recognize that $\Gamma(2)(x+1/2)^2 ye^{-y(x+1/2)}$ is the density function for a gamma random variable with $\alpha = 2$ and $\theta = (x+1/2)^{-1}$, so it would integrate to 1.

Question #205
Key: D

State#	Number	Probability of needing Therapy	Mean Number of visits E(X)	E(N)	Var(N)	Var(X)	E(S)	Var(S)
1	400	0.2	2	80	64	6	160	736
2	300	0.5	15	150	75	240	2,250	52,875
3	200	0.3	9	60	42	90	540	8,802
							2,950	62,413

$StdDev(S) = \sqrt{62,413} = 250$

$$\Pr(S > 3000) = \Pr\left(\frac{S-2950}{250} > \frac{50}{250}\right) = 1 - \Phi(0.2) = 0.42$$

The Var(X) column came from the formulas for mean and variance of a geometric distribution. Using the continuity correction, solving for Pr(S > 3000.5) is theoretically better but does not affect the rounded answer.

Question #206
Key: B

Frequency is geometric with $\beta = 2$, so $p_0 = 1/3,\ p_1 = 2/9,\ p_2 = 4/27$.

Convolutions of $f_X(x)$ needed are

x	$f(x)$	$f^{*2}(x)$
5	0.2	0
10	0.3	0.04

$$f_S(0) = 1/3 = 0.333,\ f_S(5) = (2/9)(0.2) = 0.044,\ f_S(10) = (2/9)(0.3) + (4/27)(0.04) = 0.073$$
$$E(X) = 0.2(5) + 0.3(10) + 0.5(20) = 14$$
$$E(S) = 2E(X) = 28$$
$$E[(S-15)_+] = E(S) - 5[1-F(0)] - 5[1-F(5)] - 5[1-F(10)]$$
$$= 28 - 5(1-0.333) - 5(1-0.333-0.044) - 5(1-0.333-0.044-0.073) = 18.81$$

Alternatively,

$$E[(S-15)_+] = E(S) - 15 + 15 f_S(0) + 10 f_S(5) + 5 f_S(10)$$
$$= 28 - 15 + 15(0.333) + 10(0.044) + 5(0.073) = 18.81$$

Question #207
Key: E

$$S_X(4) = 1 - \int_0^4 f_X(s)dx = 1 - \int_0^4 0.02x dx = 1 - 0.01x^2 \Big|_0^4 = 0.84$$

$$f_{Y^P}(y) = \frac{f_X(y+4)}{S_X(4)} = \frac{0.02(y+4)}{0.84} = 0.0238(y+4)^2$$

$$E(Y^P) = \int_0^6 y[0.0238(y+4)]dy = 0.0238\left(\frac{y^3}{3} + \frac{4y^2}{2}\right)\Big|_0^6 = 3.4272$$

Question #208 - DELETED

Question #209
Key: D

For any deductible *d* and the given severity distribution

$$E[(X-d)_+] = E(X) - E(X \wedge d) = 3000 - 3000\left(1 - \frac{3000}{3000+d}\right) = 3000\left(\frac{3000}{3000+d}\right) = \frac{9,000,000}{3000+d}$$

So $P_{2005} = 1.2\frac{9,000,000}{3000+600} = 3000$

Let r denote the reinsurer's deductible relative to insured losses. Thus, the reinsurer's deductible is 600 + r relative to losses. Thus

$$R_{2005} = 1.1\left(\frac{9,000,000}{3000+600+r}\right) = 0.55P_{2005} = 0.55(3000) = 1650 \Rightarrow r = 2400$$

In 2006, after 20% inflation, losses will have a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 1.2(3000) = 3600$. The general formula for expected claims with a deductible of *d* is

$$E[(X-d)_+] = 3600\left(\frac{3600}{3600+d}\right) = \frac{12,960,000}{3600+d}$$

$$P_{2006} = 1.2\frac{12,960,000}{3000+600} = 3703,\ R_{2006} = 1.1\frac{12,960,000}{3000+600+2400} = 2160,\ \frac{R_{2006}}{R_{2005}} = \frac{2160}{3703} = 0.583$$

Question #210
Key: C

Consider Disease 1 and Other Diseases as independent Poisson processes with respective lambdas 016(1/16) = 0.01 and 0.16(15/16) = 0.15. Let S denote aggregate losses from Disease 1 and T denote aggregate losses from other diseases. Let $W = S + T$.

$$E(S) = 100(0.01)(5) = 5, \quad Var(S) = 100(0.01)(50^2 + 5^2) = 2525$$
$$E(T) = 100(0.15)(10) = 150, \quad Var(T) = 100(0.15)(20^2 + 10^2) = 7500$$

If no one gets the vaccine:
$$E(W) = 5 + 150 = 155, \quad Var(W) = 2525 + 7500 = 10,025$$
$$\Phi\left(0.7\right) = 1 - 0.24, \quad A = 155 + 0.7\sqrt{10,025} = 225.08$$

If all get the vaccine, the vaccine cost = 100(0.15) = 15. Then,
$$B = 15 + 150 + 0.7\sqrt{7500} = 225.62, \quad A / B = 0.998$$

Question #211
Key: A

For the current model $f(x) = (1/4)e^{-x/4}$.

Let $g(x)$ be the new density function, which has

(i) $g(x) = c, \quad 0 \le x \le 3$

(ii) $g(x) = ke^{-x/4}, \quad x > 3$

(iii) $c = ke^{-3/4}$, since the function is continuous at $x = 3$

Since g is density function, it must integrate to 1.
$$1 = \int_0^3 c\,dx + \int_3^\infty ke^{-x/4}dx = 3c + 4ke^{-3/4} = 3c + 4c \Rightarrow c = 1/7$$
$$F(3) = \int_0^3 c\,dx = 3c = 3/7 = 0.43$$

Question #212
Key: C

Since loss amounts are uniform on (0, 10), 40% of losses are below the deductible (4), and 60% are above. Thus, claims occur at a Poisson rate $\lambda^* = 0.6(10) = 6$.

Since loss amounts were uniform on (0, 10), claims are uniform on (0, 6).

Let N = number of claims; X = claim amount; S = aggregate claims.

$E(N) = Var(N) = \lambda^* = 6$

$E(X) = (6-0)/2 = 3$

$Var(X) = (6-0)^2/12 = 3$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 6(3) + 3^2(6) = 72$

Question #213
Key: E

N	p_n	np_n	$n^2 p_n$
0	0.1	0	0
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$E(N) = 1.6$	$E(N^2) = 3.4$

$Var(N) = 3.4 - 1.6^2 = 0.84$

$E(X) = Var(X) = \lambda = 3$

$Var(S) = E(N)Var(X) + E(X)^2 Var(N) = 1.6(3) + 3^2(0.84) = 12.36$

Question #214 - DELETED

Question #215
Key: A

$E(X \mid \lambda) = Var(X \mid \lambda) = \lambda$

$\mu = v = E(\lambda) = \alpha\theta;\ a = Var(\lambda) = \alpha\theta^2;\ k = v/a = 1/\theta$

$$Z = \frac{n}{n+1/\theta} = \frac{n\theta}{n\theta+1}$$

$$0.15 = \frac{\theta}{\theta+1}(1) + \frac{1}{\theta+1}\mu = \frac{\theta+\mu}{\theta+1}$$

$$0.20 = \frac{2\theta}{2\theta+1}(2) + \frac{1}{2\theta+1}\mu = \frac{4\theta+\mu}{2\theta+1}$$

From the first equation,

$0.15\theta + 0.15 = \theta + \mu$ and so $\mu = 0.15 - 0.85\theta$

Then the second equation becomes

$0.4\theta + 0.2 = 4\theta + 0.15 - 0.85\theta$

$0.05 = 2.75\theta;\ \theta = 0.01818$

Question #216 - DELETED

Question #217 - DELETED

Question #218
Key: E

$$f(x) = -S'(x) = \frac{4x\theta^4}{(\theta^2+x^2)^3}$$

$$L(\theta) = f(2)f(4)S(4) = \frac{4(2)\theta^4}{(\theta^2+2^2)^3}\frac{4(4)\theta^4}{(\theta^2+4^2)^3}\frac{\theta^4}{(\theta^2+4^2)^2} = \frac{128\theta^{12}}{(\theta^2+4)^3(\theta^2+16)^5}$$

$l(\theta) = \ln 128 + 12\ln\theta - 3\ln(\theta^2+4) - 5\ln(\theta^2+16)$

$$l'(\theta) = \frac{12}{\theta} - \frac{6\theta}{\theta^2+4} - \frac{10\theta}{\theta^2+16} = 0;\ 12(\theta^4+20\theta^2+64) - 6(\theta^4+16\theta^2) - 10(\theta^4+4\theta^2) = 0$$

$0 = -4\theta^4 + 104\theta^2 + 768 = \theta^4 - 26\theta^2 - 192$

$$\theta^2 = \frac{26 \pm \sqrt{26^2+4(192)}}{2} = 32;\ \theta = 5.657$$

Question #219
Key: A

$$E(X\mid\theta)=\int_0^\theta x\frac{2x}{\theta^2}dx=\frac{2\theta}{3};Var(X\mid\theta)=\int_0^\theta x^2\frac{2x}{\theta^2}dx-\frac{4\theta^2}{9}=\frac{\theta^2}{2}-\frac{4\theta^2}{9}=\frac{\theta^2}{18}$$

$$\mu=(2/3)E(\theta)=(2/3)\int_0^1 4\theta^4 d\theta=8/15$$

$$EPV=v=(1/18)E(\theta^2)=(1/18)\int_0^1 4\theta^5 d\theta=1/27$$

$$VHM=a=(2/3)^2Var(\theta)=(4/9)\left[4/6-(4/5)^2\right]=8/675$$

$$k=\frac{1/27}{8/675}=25/8;\ Z=\frac{1}{1+25/8}=8/33$$

Estimate is $(8/33)(0.1)+(25/33)(8/15)=0.428.$

Question #220 - DELETED

Question #221 - DELETED

Question #222
Key: E

For the Poisson distribution, the mean, λ, is estimated as 230/1000 = 0.23.

# of Days	Poisson Probability	Expected # of Workers	Observed # of Workers	Chi-square
0	0.794533	794.53	818	0.69
1	0.182743	182.74	153	4.84
2	0.021015	21.02	25	0.75
3 or more	0.001709	1.71	4	3.07
Total			1000	9.35

The chi-square distribution has 2 degrees of freedom because there are four categories and the Poisson parameter is estimated (d.f. = 4 – 1 – 1 = 2). The critical values for a chi-square test with two degrees of freedom are shown in the following table.

Significance Level	Critical Value
10%	4.61
5%	5.99
2.5%	7.38
1%	9.21

9.35 is greater than 9.21 so the null hypothesis is rejected at the 1% significance level.

Question #223
Key: D

$EPV = \hat{v} = \dfrac{25(480-472.73)^2 + 30(466.67-472.73)^2}{2-1} = 2423.03$ where 480 = 12,000/25, 466.67 = 14,000/30, and 472.73 = 26,000/55.

$k = 2423.03 / 254 = 9.54; Z = \dfrac{55}{55+9.54} = 0.852$

Question #224 - DELETED

Question #225
Key: C

The quantity of interest is $P = \Pr(X \le 5000) = \Phi\left(\dfrac{\ln 5000 - \mu}{\sigma}\right)$. The point estimate is

$\Phi\left(\dfrac{\ln 5000 - 6.84}{1.49}\right) = \Phi(1.125) = 0.87$.

For the delta method:

$\dfrac{\partial P}{\partial \mu} = \dfrac{-\phi(1.125)}{1.49} = -0.1422; \dfrac{\partial P}{\partial \sigma} = \dfrac{-1.125\phi(1.125)}{1.49} = -0.1600$ where $\phi(z) = \dfrac{1}{\sqrt{2\pi}} e^{-z^2/2}$.

Then the variance of $\hat{P}$ is estimated as $(-0.1422)^2 0.0444 + (-0.16)^2 0.0222 = 0.001466$ and the lower limit is $P_L = 0.87 - 1.96\sqrt{0.001466} = 0.79496$.

Question #226
Key: A

$\Pr(\theta = 0.1 | X_1 = 1) = \dfrac{\Pr(X_1 = 1 | \theta = 0.1)\Pr(\theta = 0.1)}{\Pr(X_1 = 1 | \theta = 0.1)\Pr(\theta = 0.1) + \Pr(X_1 = 1 | \theta = 0.3)\Pr(\theta = 0.3)}$

$= \dfrac{0.1(0.8)}{0.1(0.8) + 0.3(0.2)} = \dfrac{4}{7}$

Then,

$E(X_2 | \theta = 0.1) = 0(0.2) + 1(0.1) + 2(0.7) = 1.5$

$E(X_2 | \theta = 0.3) = 0(0.6) + 1(0.3) + 2(0.1) = 0.5$

$E(X_2 | X_1 = 1) = (1.5)(4/7) + (0.5)(3/7) = 1.071$

Question #227 - DELETED

Question #228 - DELETED

Question #229
Key: A

$$\ln f(x) = \ln\theta - 2\ln(\theta + x)$$

$$\frac{\partial \ln f(x)}{\partial\theta} = \frac{1}{\theta} - \frac{2}{\theta + x}$$

$$\frac{\partial^2 \ln f(x)}{\partial\theta^2} = -\frac{1}{\theta^2} + \frac{2}{(\theta + x)^2}$$

$$E\left[\frac{\partial^2 \ln f(x)}{\partial\theta^2}\right] = -\frac{1}{\theta^2} + \int_0^\infty \frac{2\theta}{(\theta + x)^4}\,dx = -\frac{1}{\theta^2} + \left[-\frac{2\theta}{3(\theta + x)^3}\right]\Bigg|_0^\infty = -\frac{1}{\theta^2} + \frac{2}{3\theta^2} = -\frac{1}{3\theta^2}$$

$$I(\theta) = \frac{n}{3\theta^2}; \quad Var = \frac{3\theta^2}{n}$$

Question #230
Key: B

$$\mu = E[E(X \mid \lambda)] = E(\lambda) = 1(0.9) + 10(0.09) + 20(0.01) = 2$$

$$EPV = v = E[Var(X \mid \lambda)] = E(\lambda) = 2$$

$$VHM = a = Var[E(X \mid \lambda)] = Var(\lambda) = 1(0.9) + 100(0.09) + 400(0.01) - 2^2 = 9.9$$

$$Z = \frac{1}{1 + 2/9.9} = 0.83193; \quad 11.983 = 0.83193x + 0.16807(2); \quad x = 14$$

Question #231 DELETED

Question #232 - DELETED

Question #233
Key: A

$$\hat{\mu} = \bar{x} = 12/60 = 0.2,\ EVPV = \hat{v} = \bar{x} = 0.2$$

$$VHM = \hat{a} = \frac{10(0.4-0.2)^2 + 20(0.25-0.2)^2 + 30(0.1-0.2)^2 - (3-1)(0.2)}{60 - \dfrac{10^2 + 20^2 + 30^2}{60}} = 0.009545$$

$$\hat{k} = 20.9524; \quad Z = \frac{10}{10 + 20.9524} = 0.323$$

Question #234 - DELETED

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Question #235
Key: C

$l(\tau,\theta)=\sum_{j=1}^{5}\ln f(x_j)=\sum_{j=1}^{5}\ln\tau+(\tau-1)\ln x_j-\tau\ln\theta-(x_j/\theta)^{\tau}$. Under the null hypothesis it is $l(2,\theta)=\sum_{j=1}^{5}\ln 2+\ln x_j-2\ln\theta-(x_j/\theta)^2$. Inserting the maximizing value of 816.7 for θ gives –35.28. The likelihood ratio test statistic is 2(– 33.05 + 35.28) = 4.46. There is one degree of freedom. At a 5% significance level the critical value is 3.84 and at a 2.5% significance level it is 5.02.

Question #236
Key: C

It is given that n = 4, v = 8, and Z = 0.4. Then, $0.4=\dfrac{4}{4+\dfrac{8}{a}}$ which solves for a = 4/3. For the covariance,

$$\begin{aligned}Cov(X_i,X_j)&=E(X_iX_j)-E(X_i)E(X_j)\\&=E[E(X_iX_j\,|\,\theta)]-E[E(X_i\,|\,\theta)]E[E(X_j\,|\,\theta)]\\&=E[\mu(\theta)^2]-E[\mu(\theta)]^2=Var[\mu(\theta)]=a=4/3.\end{aligned}$$

Question #237 – DELETED

Question #238 - DELETED

Question #239 - DELETED

Question #240
Key: D

$\bar{x}=\dfrac{5000(0)+2100(1)+750(2)+100(3)+50(4)}{8000}=0.5125$ and

$s^2=\dfrac{5000(0.5125)^2+2100(0.4875)^2+750(1.4875)^2+100(2.4875)^2+50(3.4875)^2}{7999}=0.5874$.

Then, $\hat{\mu}=\hat{v}=\bar{x}=0.5125$ and $\hat{a}=s^2-\bar{x}=0.0749$. The credibility factor is

$Z=\dfrac{1}{1+0.5125/0.0749}=0.1275$ and the estimate is 0.1275(1) + 0.8725(0.5125) = 0.5747.

Question #241
Key: B

$s = F_n(3000) = 4/8 = 0.5$ because for the *p*-*p* plot the denominator is *n*+1.
$t = F(3000) = 1 - e^{-3000/3300} = 0.59711$. For the difference plot, *D* uses a denominator of *n* and so *D* = 4/7 – 0.59711 = –0.02568 and the answer is 0.5 – 0.59711 + 0.02568 = –0.071.

Question #242
Key: B

$\pi(q \mid 2,2) \propto f(2 \mid q) f(2 \mid q) \pi(q) = q(q)(q^2/0.039) \propto q^4$. Because $\int_{0.2}^{0.5} q^4 dq = 0.006186$,
$\pi(q \mid 2,2) = q^4/0.006186$. Given *q*, the expected number of claims is
$E(N \mid q) = 0(0.1) + 1(0.9 - q) + 2q = 0.9 + q$. The Bayesian estimate is
$E(N \mid 2,2) = \int_{0.2}^{0.5} (0.9+q) \dfrac{q^4}{0.006186} dq = 1.319.$

Question #243 - DELETED

Question #244
Key: A

A is false because the test works best when the expected number of observations is about the same from interval to interval.

Question #245
Key: E

$$n\lambda \ge \lambda_0 \left[1 + \left(\frac{\sigma_Y}{\theta_Y}\right)^2\right]; \theta_Y = \alpha\theta = 10,000\alpha; \sigma_Y^2 = \alpha\theta^2 = 10^8\alpha$$

$$n\lambda \ge \left(\frac{1.96}{0.1}\right)^2 \left[1 + \frac{10^8\alpha}{10^8\alpha^2}\right] = 384.16(1+\alpha^{-1})$$

Because α is needed, but not given, the answer cannot be determined from the information given.

Question #246 - DELETED

Question #247
Key: D

Let E be the even of having 1 claim in the first four years. In four years, the total number of claims is Poisson(4λ).

$$\Pr(Type\,I \mid E) = \frac{\Pr(E \mid Type\,I)\Pr(Type\,I)}{\Pr(E)} = \frac{e^{-1}(0.05)}{\Pr(E)} = \frac{0.01839}{\Pr(E)} = 0.14427$$

$$\Pr(Type\,II \mid E) = \frac{e^{-2}(2)(0.2)}{\Pr(E)} = \frac{0.05413}{\Pr(E)} = 0.42465$$

$$\Pr(Type\,III \mid E) = \frac{e^{-4}(4)(0.75)}{\Pr(E)} = \frac{0.05495}{\Pr(E)} = 0.43108$$

$$Note: \Pr(E) = 0.01839 + .05413 + .05495 = 0.12747$$

The Bayesian estimate of the number of claims in Year 5 is:
0.14427(0.25) + 0.42465(0.5) + 0.43108(1) = 0.67947.

Question #248 - DELETED

Question #249 - DELETED

Question #250
Key: A

The density function is $f(x) = \theta x^{-2} e^{-\theta/x}$ and the likelihood function is

$$L(\theta) = \theta(186^{-2})e^{-\theta/186}\theta(91^{-2})e^{-\theta/91}\theta(66^{-2})e^{-\theta/66}(e^{-\theta/60})^{7} \propto \theta^{3}e^{-0.148184\theta}$$

$$l(\theta) = \ln L(\theta) = 3\ln(\theta) - 0.148184\theta$$

$$l'(\theta) = 3\theta^{-1} - 0.148184 = 0$$

$$\theta = 3/0.148184 = 20.25.$$

The mode is $\theta/2 = 20.25/2 = 10.125$.

Question #251
Key: D

We have $\mu(\theta) = 4\theta$ and $\mu = 4E(\theta) = 4(600) = 2400$. The average loss for Years 1 and 2 is 1650 and so 1800 = Z(1650) + (1 – Z)(2400) which gives Z = 0.8. Because there were two years, Z = 0.8 = 2/(2 + k) which gives k = 0.5.

For three years, the revised value is Z = 3/(3 + 0.5) = 6/7 and the revised credibility estimate (using the new sample mean of 2021), (6/7)(2021) + (1/7)(2400) = 2075.14.

Question #252 - DELETED

Question #253
Key: E

$S_m \mid Q \sim bin(m,Q)$ and $Q \sim beta(1,99)$. Then

$E(S_m) = E[E(S_m \mid Q)] = E(mQ) = m\dfrac{1}{1+99} = 0.01m$. For the mean to be at least 50, *m* must be at least 5,000.

Question #254
Key: D

The posterior distribution is

$\pi(\lambda \mid data) \propto (e^{-\lambda})^{90}(\lambda e^{-\lambda})^{7}(\lambda^{2}e^{-\lambda})^{2}(\lambda^{3}e^{-\lambda})\dfrac{\lambda^{4}e^{-50\lambda}}{\lambda} = \lambda^{17}e^{-150\lambda}$ which is a gamma distribution with parameters 18 and 1/150. For one risk, the estimated value is the mean, 18/150. For 100 risks it is 100(18)/150 = 12.

Alternatively,

The prior distribution is gamma with $\alpha = 4$ and $\beta = 50$. The posterior will continue to be gamma, with $\alpha' = \alpha + \text{no. of claims} = 4 + 14 = 18$ and
$\beta' = \beta + \text{no. of exposures} = 50 + 100 = 150$. Mean of the posterior is
$\alpha / \beta = 18/150 = 0.12$.
Expected number of claims for the portfolio is 0.12(100) = 12.

Question #255 - DELETED

Question #256
Key: B

$$L(q) = \left[\binom{2}{0}(1-q)^{2}\right]^{5000}\left[\binom{2}{1}q(1-q)\right]^{5000} = 2^{5000}q^{5000}(1-q)^{15000}$$

$$l(q) = 5000\ln(2) + 5000\ln(q) + 15000\ln(1-q)$$

$$l'(q) = 5000q^{-1} - 15000(1-q)^{-1} = 0$$

$$\hat{q} = 0.25$$

$$l(0.25) = 5000\ln(2) + 5000\ln(0.25) + 15000\ln(0.75) = -7780.97.$$

Question #257
Key: C

The estimate of the overall mean, μ, is the sample mean, per vehicle, which is 7/10 = 0.7. With the Poisson assumption, this is also the estimate of v = EPV. The means for the two insureds are 2/5 = 0.4 and 5/5 = 1.0. The estimate of a is the usual non-parametric estimate,

$$\text{VHM} = \hat{a} = \frac{5(0.4-0.7)^2 + 5(1.0-0.7)^2 - (2-1)(0.7)}{10 - \frac{1}{10}(25+25)} = 0.04$$

Then, k = 0.7/0.04 = 17.5 and so Z = 5/(5+17.5) = 2/9. The estimate for insured A is (2/9)(0.4) + (7/9)(0.7) = 0.6333.

Question #258 - DELETED

Question #259
Key: B

The estimator of the Poisson parameter is the sample mean. Then,

$$E(\hat{\lambda}) = E(\bar{X}) = \lambda$$
$$Var(\hat{\lambda}) = Var(\bar{X}) = \lambda / n$$
$$c.v. = \sqrt{\lambda / n} / \lambda = 1/\sqrt{n\lambda}$$

It is estimated by $1/\sqrt{n\lambda} = 1/\sqrt{39} = 0.1601.$

Question #260
Key: E

$$\begin{aligned}\Pr\left(\theta = 8 \mid X_1 = 5\right) &= \frac{\Pr(X_1 = 5 \mid \theta = 8)\Pr(\theta = 8)}{\Pr(X_1 = 5 \mid \theta = 8)\Pr(\theta = 8) + \Pr(X_1 = 5 \mid \theta = 2)\Pr(\theta = 2)} \\ &= \frac{0.125e^{-5(0.125)}(0.8)}{0.125e^{-5(0.125)}(0.8) + 0.5e^{-5(0.5)}(0.2)} = 0.867035.\end{aligned}$$

Then,

$$E(X_2 \mid X_1 = 5) = E(\theta \mid X_1 = 5) = 0.867035(8) + 0.132965(2) = 7.202.$$

Question #261 - DELETED

Question #262
Key: D

$$L(\omega)=\frac{\frac{1}{\omega}\frac{1}{\omega}\frac{1}{\omega}\left(\frac{\omega-4-p}{\omega}\right)^2}{\left(\frac{\omega-4}{\omega}\right)^5}=\frac{(\omega-4-p)^2}{(\omega-4)^5}$$

$$l(\omega)=2\ln(\omega-4-p)-5\ln(\omega-4)$$

$$l'(\omega)=\frac{2}{\omega-4-p}-\frac{5}{\omega-4}=0$$

$$0=l'(29)=\frac{2}{25-p}-\frac{5}{25}$$

$p=15.$

The denominator in the likelihood function is *S*(4) to the power of five to reflect the fact that it is known that each observation is greater than 4.

Question #263
Key: B

$$\mu(\lambda)=v(\lambda)=\lambda$$

$$\mu=v=E(\lambda)=0.1\Gamma(1+1/2)=0.088623$$

$$a=Var(\lambda)=(0.1)^2\Gamma(1+2/2)-0.088623^2=0.002146$$

$$Z=\frac{500}{500+0.088623/0.002146}=0.92371.$$

The estimate for one insured for one month is 0.92371(35/500) + 0.07629(0.088623) = 0.07142. For 300 insureds for 12 months it is (300)(12)(0.07142) = 257.11.

Question #264 - DELETED

Question #265 - DELETED

Question #266 - DELETED

Question #267
Key: E

$$\Pr\left(\lambda=1 \mid X_1=r\right)=\frac{\Pr(X_1=r \mid \lambda=1)\Pr(\lambda=1)}{\Pr(X_1=r \mid \lambda=1)\Pr(\lambda=1)+\Pr(X_1=r \mid \lambda=3)\Pr(\lambda=3)}$$

$$=\frac{\frac{e^{-1}}{r!}(0.75)}{\frac{e^{-1}}{r!}(0.75)+\frac{e^{-3}3^r}{r!}(0.25)}=\frac{0.2759}{0.2759+0.1245(3^r)}.$$

Then,

$$2.98=\frac{0.2759}{0.2759+0.1245(3^r)}(1)+\frac{0.1245(3^r)}{0.2759+0.1245(3^r)}(3)=\frac{0.2759+0.3735(3^r)}{0.2759+0.1245(3^r)}.$$

Rearrange to obtain

$$0.82218+0.037103(3^r)=0.2759+0.03735(3^r)$$

$$0.54628=0.00025(3^r)$$

$$r=7.$$

Because the risks are Poisson:

$$\mu=v=E(\lambda)=0.75(1)+0.25(3)=1.5$$

$$a=Var(\lambda)=0.75(1)+0.25(9)-2.25=0.75$$

$$Z=\frac{1}{1+1.5/0.75}=1/3$$

and the estimate is (1/3)(7) + (2/3)(1.5) = 3.33.

Question #268 - DELETED

Question #269 - DELETED

Question #270
Key: C

The sample means are 3, 5, and 4 and the overall mean is 4. Then,

$$\hat{v}=\frac{1+0+0+1+0+0+1+1+1+1+1+1}{3(4-1)}=\frac{8}{9}$$

$$\hat{a}=\frac{(3-4)^2+(5-4)^2+(4-4)^2}{3-1}-\frac{8/9}{4}=\frac{7}{9}=0.78.$$

Question #271 - DELETED

Question #272
Key: C

$\pi(q\,|\,2) = 6q^2(1-q)^2 6q(1-q) \propto q^3(1-q)^3$

The mode can be determined by setting the derivative equal to zero.

$\pi'(q\,|\,2) \propto 3q^2(1-q)^3 - 3q^3(1-q)^2 = 0$

$(1-q) - q = 0$

$q = 0.5.$

Question #273
Key: B

For the severity distribution the mean is 5,000 and the variance is $10,000^2/12$. For credibility based on accuracy with regard to the number of claims,

$2000 = \left(\dfrac{z}{0.03}\right)^2, \quad z^2 = 1.8$

Where z is the appropriate value from the standard normal distribution. For credibility based on accuracy with regard to the total cost of claims, the number of claims needed is

$\dfrac{z^2}{0.05^2}\left(1 + \dfrac{10000^2/12}{5000^2}\right) = 960.$

Question #274 - DELETED

Question #275 - DELETED

Question #276
Key: B

$L(\theta) = \left(1 - \dfrac{\theta}{10}\right)^9 \left(\dfrac{\theta}{10} - \dfrac{\theta}{25}\right)^6 \left(\dfrac{\theta}{25}\right)^5 \propto (10-\theta)^9\theta^{11}$

$l(\theta) = 9\ln(10-\theta) + 11\ln(\theta)$

$l'(\theta) = -\dfrac{9}{10-\theta} + \dfrac{11}{\theta} = 0$

$11(10-\theta) = 9\theta$

$110 = 20\theta$

$\theta = 110/20 = 5.5.$

Question #277
Key: A

The maximum likelihood estimate is $\hat{\theta} = \bar{x} = 1000$. The quantity to be estimated is $S(\theta) = \exp(-1500/\theta)$ and $S'(\theta) = 1500\theta^{-2}\exp(-1500/\theta)$. For the delta method,
$Var[S(\hat{\theta})] \cong [S'(\hat{\theta})]^2 Var(\hat{\theta}) = [1500(1000)^{-2}\exp(-1500/1000)]^2(1000^2/6) = 0.01867.$
This is based on $Var(\hat{\theta}) = Var(\bar{X}) = Var(X)/n = \theta^2/n$.

Question #278 - DELETED

Question #279
Key: B

Pays 80% of loss over 20, with cap of payment at 60, hence u = 60/0.8 + 20 = 95.

$$E(Y \text{ per loss}) = \alpha[E(X \wedge 95) - E(X \wedge 20)] = 0.8\left[\int_0^{95} S(x)dx - \int_0^{20} S(x)dx\right]$$

$$= 0.8\int_{20}^{95} S(x)dx = 0.8\int_{20}^{95}\left(1 - \frac{x^2}{10{,}000}\right)dx = 0.8\left(x - \frac{x^3}{30{,}000}\right)\Bigg|_{20}^{95} = 37.35$$

$$E(Y \text{ per payment}) = \frac{E(Y \text{ per loss})}{1 - F(20)} = \frac{37.35}{0.96} = 38.91$$

Question #280
Key: D

Let S = aggregate claims, I_5 = claims covered by stop loss
$E(I_5) = E(S) - 5 - 5\Pr(S=0)$
$E(S) = 5[0.6(5) + 0.4k] = 15 + 2k$
$\Pr(S=0) = e^{-5}$
$E(I_5) = 15 + 2k - 5 - 5e^{-5} = 28.03$
$10.034 + 2k = 28.03$
$k = 9$

Question #281 - DELETED

Question #282
Key: A

Let S = aggregate losses, X = severity
Since the frequency is Poisson,
$Var(S) = \lambda E\left(X^2\right)$
$E(X^2) = \dfrac{2^2\Gamma(3)\Gamma(1)}{\Gamma(3)} = 4$ (table lookup)
$Var(S) = 3(4) = 12$
You would get the same result if you used
$Var(S) = E(N)Var(X) + Var(N)E(X)^2$

Question #283
Key: D

For each member $P(z) = \left[1 - 1.5(z-1)\right]^{-1}$
so for family of 4, $P(z) = \left[1 - 1.5(z-1)\right]^{-4}$ which is negative binomial with $\beta = 1.5, r = 4$

k	p_k
0	0.026
1	0.061
2	0.092
3+	0.821

$E(N \wedge 3) = 0(0.026) + 1(0.061) + 2(0.092) + 3(0.821) = 2.71$
$E(N) - E(N \wedge 3) = 6 - 2.71 = 3.29$
3.29(100 per visit) = 329

Alternatively, without using probability generating functions, a geometric distribution is a special case of the negative binomial with r = 1. Summing four independent negative binomial distributions, each with $\beta = 1.5, r = 1$ gives a negative binomial distribution with $\beta = 1.5, r = 4$. Then continue as above.

Question #284
Key: E

$$E(X \wedge 2) = 1f(1) + 2[1 - F(1)] = 1f(1) + 2[1 - f(0) - f(1)]$$
$$= 1(3e^{-3}) + 2(1 - e^{-3} - 3e^{-3}) = 2 - 5e^{-3} = 1.75$$

Cost per loss with deductible is
$E(X) - E(X \wedge 2) = 3 - 1.75 = 1.25$

Cost per loss with coinsurance is $\alpha E(X) = 3\alpha$
Equating cost: $3\alpha = 1.25 \Rightarrow \alpha = 0.42$

Question #285
Key: A

Let N be the number of clubs accepted
X be the number of members of a selected club
S be the total persons appearing

N is binomial with m = 1000 q = 0.20
$E(N) = 1000(0.20) = 200, \quad Var(N) = 1000(0.20)(0.80) = 160$
$E(S) = E(N)E(S) = 200(20) = 4000$
$Var(S) = E(N)Var(S) + E(X)^2 Var(N) = 200(20) + 20^2(160) = 68,000$

Budget $= 10E(S) + 10\sqrt{Var(S)} = 10(4000) + 10\sqrt{68,000} = 42,610$

Question #286
Key: C

Insurance pays 80% of the portion of annual claim between 6,000 and 1,000, and 90% of the portion of annual claims over 14,000.

The 14,000 breakpoint is where Michael reaches 10,000 that he has paid:

1000 = deductible
1000 = 20% of costs between 1000 and 6000
8000 = 100% of costs between 14,000 and 6,000

$$E(X \wedge x) = \theta\left(1 - \frac{\theta}{x+\theta}\right) = \frac{5000x}{x+5000}$$

x	$E(X \wedge x)$
1000	833.33
6000	2727.27
14000	3684.21
∞	5000

$$\begin{aligned}&0.80[E(X \wedge 6000) - E(X \wedge 1000)] + 0.90[E(X) - \mathrm{E}(X \wedge 14000)]\\&\quad = 0.80[2727,27 - 833.33] + 0.90[5000 - 3684.21]\\&\quad = 1515.15 + 1184.21 = 2699.36\end{aligned}$$

Question #287
Key: D

We have the following table:

Item	Dist	$E(\ )$	$Var(\ )$
Number claims	$NB(16,6)$	16(6) = 96	16(6)(7) = 672
Claims amounts	$U(0,8)$	(8 – 0)/2 = 4	$(8-0)^2/12 = 5.33$
Aggregate		4(96) = 384	$96(5.33) + 672(4^2) = 11,264$

Premium $= E(S) + 1.645\sqrt{Var(S)} = 384 + 1.645\sqrt{11,264} = 559$

Question #288
Key: E

With probability p, $\Pr(N=2) = 0.5^2 = 0.25$. With probability $(1 - p)$,

$\Pr(\mathrm{N{=}2}) = \binom{4}{2}0.5^4 = 0.375$. $\Pr(N=2) = p(0.25) + (1-p)(0.375) = 0.375 - 0.125p$

Question #289
Key: D

600 can be obtained only 2 ways, from $500+100$ or from 6(100).

Since $\lambda = 5$ and $p(100) = 0.8$, $p(500) = 0.16$.

$$\Pr\left(6 \text{ claims for } 100\right) = \frac{e^{-5}5^6}{6!}\left(0.8\right)^6 = 0.03833 \text{ or } 3.83\%$$

$$\Pr\left(\overset{\text{2 claims}}{500+100}\right) = \frac{e^{-5}5^2}{2!}\left[(0.8)^1(0.16)^1(2)\right] = 0.02156 = 2.16\%$$

The factor of 2 inside the bracket is because you could get a 500 then 100 or you could get a 100 then 500. Total is 3.83% + 2.16% = 5.99%.

Question #290 - DELETED

Question #291 - DELETED

Question #292 - DELETED

Question #293 - DELETED

Question #294 - DELETED

Question #295 - DELETED

Question #296 - DELETED

Question #297 - DELETED

Question #298 - DELETED

Question #299 - DELETED

Question #300 - DELETED

Question #301 - DELETED

Question #302 - DELETED

Question #303 - DELETED

Question #304 - DELETED

Question #305 - DELETED

Question #306
Key: A

(This question was formerly Question 266.) The deduction to get the SBC is $(r/2)\ln(n) = (r/2)\ln(260) = 2.78r$ where r is the number of parameters. The SBC values are then –416.78, –417.56, –419.34, –420.12, and –425.68. The largest value is the first one, so model I is to be selected.

Question #307
Key: D

(This question is effective with the October 2016 syllabus.) The deduction to get the AIC is r, where r is the number of parameters. The AIC values are then –415, –414, –414, –413, and –415. The largest value is the fourth one, so model IV is to be selected.

Question #308
Key: D

At 250 the payment is 0. At 1000 the payment is 1000. Interpolating:
$$\frac{700-250}{1000-250}=\frac{x-0}{1000-0}\Rightarrow x=450(1000)/750=600.$$

Question #309
Key: C

$F(x)=\int_0^x 2y\,dy=x^2$. Let C be a random claim payment. Then C = 0 if $X < d$ and $C = X - d$ if $X \geq d$. Then,
$$P(C<0.5)=0.64$$
$$P(C\geq 0.5)=0.36$$
$$P(X-d\geq 0.5)=0.36$$
$$P(X\geq 0.5+d)=0.36$$
$$F(0.5+d)=0.64$$
$$(0.5+d)^2=0.64$$
$$0.5+d=0.8$$
$$d=0.3$$

Question #310
Key: B

With no deductible, the loss cost is proportional to 598,500.
With a 100 deductible it is proportional (in the same proportion) to 598,500 – 58,500 – 1000(100) = 440,000. With a 250 deductible it is 598,500 – 58,500 – 70,000 – 600(250) = 320,000. The reduction is 120,000/440,000 = 0.273 = 27%.

Question #311
Key: A

At the time of the loss the coverage is 150,000/250,000 = 60% < 80%. Then the benefit payment is $\min\left\{150,000, \frac{150,000}{0.8(250,000)}20,000\right\}=15,000$.

Question #312
Key: D

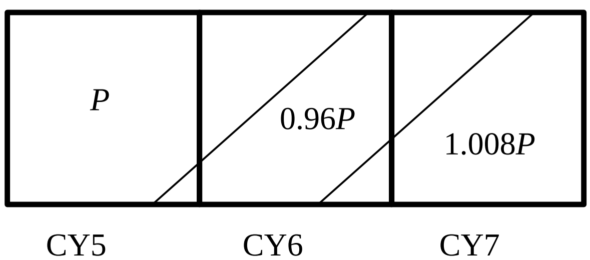
$$\min\left\{70,000, \frac{70,000}{0.8(100,000)}(20,000-200)\right\} = 17,325$$

Question #313
Key: E

The payment is 175,000 + 45,000 + (20,000 – 1,000) = 239,000. Note that the legal fees do not count against the liability limit.

Question #314
Key: C

Let P be the premium after the July 1, CY3 rate change. On November 15, CY5 the premium is $0.96P$ and on October 1, CY6 it becomes $1.05(0.96)P = 1.008P$. The relevant parallelogram is:



The upper left triangle for CY6 has area $(1/2)(7/8)^2 = 49/128$ and the lower right triangle has area $(1/2)(1/4)^2 = 4/128$. The weighted average is $[49 + 4(1.008) + 75(0.960)]P/128 = 0.9768P$. The current premium is 9200(1.008)/(0.9768) = 9494.

Question #315
Key: C

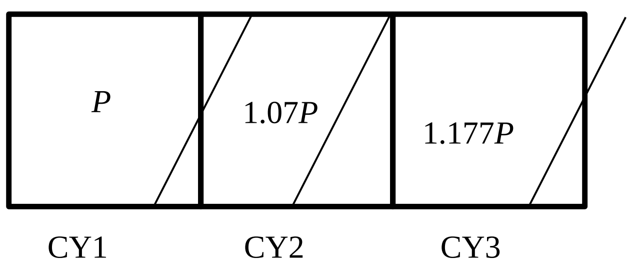
The credibility factor is $Z = \sqrt{390/1082} = 0.6$. Prior to applying credibility, the indicated relativity is $0.52\frac{360/450}{1250/1680} = 0.559$. The credibility-weighted relativity is 0.6(0.559) + 0.4(0.52) = 0.5434.

Question #316
Key: D

Trend periods for losses: Average accident date in experience period is July 1, AY8 and July 1, AY9, respectively. New rates will be in effect from July 1, CY10 through June 30, CY12, with an average accident date of July 1, CY11. Trend period for losses are 3 years for AY8 and 2 years for AY9. Ultimate losses trended and developed are, AY8: $2260(1.07^3)(1.08) = 2990$, and AY9: $2610(1.07^2)(1.18) = 3526$. Weighted average loss ratio = 0.4(2990/4252) + 0.6(3526/5765) = 0.648. Required portfolio-wide rate change = 0.648/0.657 – 1 = –1.4%.

Question #317
Key: E

Let P be the premium prior to the October 1, CY1 rate change. After the change, the premium is $1.07P$ and on July 1, CY2 it becomes $1.10(1.07)P = 1.177P$. The relevant parallelogram is:



The upper left triangle for CY2 has area (1/2)(1/4)(1/2) = 1/16 and the lower right triangle has area (1/2)(1/2)(1) = 4/16. The weighted average is $[1 + 4(1.177) + 11(1.07)]P/16 = 1.092375P$. The factor is (1.177×.94)/(1.092375) = 1.0128.

Question #318
Key: B

The year-to-year development factors are 12-24: 30,800/15,400 = 2; 24-36: 29,000/20,000 = 1.45; and 36-48: 16,200/14,100 = 1.149. Then the factor for 24-48 is 1.45(1.149) = 1.666 and for 12-48 is 2(1.666) = 3.332. The expected ultimate losses are AY6: 20,000(0.85) = 17,000; AY7: 21,000(0.91) = 19,110; and AY8: 22,000(0.88) = 19,360. The B-F reserves are 17,000(1 – 1/1.149) = 2205, 19,110(1 – 1/1.666) = 7639, and 19,360(1 – 1/3.332) = 13,550. The total is 23,394.

Question #319
Key: C

The completed triangle values are AY6: 48 – 71,000(1.15) = 81,650; AY7: 36: 65,000(1.2) = 78,000 and 48: 78,000(1.15) = 89,700; and AY8: 24 – 35,000(2) = 70,000, 36 – 70,000(1.2) = 84,000, and 48 – 84,000(1.15) = 96,600. The undiscounted reserve is:
(81,650 – 71,000) + (89,700 – 65,000) + (96,600 – 35,000) = 96,950.

The increments are AY6: 10,650; AY7: 13,000 and 11,700; and AY8: 35,000, 14,000, and 12,600. By development month the totals are next 12 months: 58,650, 12-24 months: 25,700, and 24-36 months: 12,600. The discounted value is $58{,}650/1.05^{0.5} + 25{,}700/1.05^{1.5} + 12{,}600/1.05^{2.5} = 92{,}276$.

The ratio is 92,276/96,950 = 0.95.

Question #320
Key: D

The development factors are:

$$0-1: \frac{1}{3}\left(\frac{7,900}{2,500}+\frac{8,700}{2,800}+\frac{7,500}{2,500}\right)=3.089 \quad 1-2: \frac{1}{3}\left(\frac{12,900}{8,700}+\frac{10,700}{7,500}+\frac{8,800}{6,400}\right)=1.428$$

$$2-3: \frac{1}{3}\left(\frac{12,600}{10,700}+\frac{10,200}{8,800}+\frac{8,800}{7,300}\right)=1.181 \quad 3-4: \frac{1}{2}\left(\frac{11,500}{10,200}+\frac{9,800}{8,800}\right)=1.1205 \quad 4-5: 1$$

The cumulative factors are 0-5: 5.8372, 1-5: 1.8897, 2-5: 1.3233, 3-5: 1.1205; 4-5: 1.
The B-F reserve is [25,000(1 – 1/1.1205) + 26,000(1 – 1/1.3233) + 27,000(1 – 1/1.8897) + 28,000(1 – 1/5.8372)](0.55) = 24,726.

Question #321
Key: E

CY1: ELR = 60%, earned premium = 1,000,000, expected losses = 0.6(1,000,000) = 600,000. Cumulative development factor (CDF) = (1.05)(1.057)(1.126) = 1.250. Reserve using BF method = 600,000(1 – 1/1.250) = 120,000.
CY2: ELR = 62%, earned premium = 1,200,000, expected losses = 0.62(1,200,000) = 744,000. CDF = (1.05)(1.057)(1.126)(1.336) = 1.670. Reserve using BF method = 744,000(1 – 1/1.670) = 298,491.
CY3: ELR = 64%, earned premium = 1,440,000, expected losses = 0.64(1,440,000) = 921,600. CDF = (1.05)(1.057)(1.126)(1.336)(1.5) = 2.504. Reserve using BF method = 921,600(1 – 1/2.504) = 553,549.
Total reserve = 120,000 + 298,491 + 553,549 = 972,040.

Question #322
Key: E

The probabilities for the respective intervals are 0.358, 0.403, 0.118, 0.051, 0.026, 0.028, and 0.016. The expected loss at the basic limit is 0.358(300) + 0.403(8,200) + 0.118(47,500) + 0.121(100,000) = 21,117. The expected loss at the increased limit is 0.358(300) + 0.403(8,200) + 0.118(47,500) + 0.051(145,000) + 0.026(325,000) + 0.028(650,000) + 0.016(1,000,000) = 59,062. The ILF is 59,062/21,117 = 2.797.

Question #323
Key: E

The increased limits factor is 25,000,000/14,000,000 and so the pure premium is 240(25/14) = 428.57. The fixed expense stays at 30. The variable expense is 30/300 = 0.1 of the rate. So the rate is the solution to 0.1r + 30 + 428.57 = r and thus r = 458.57/0.9 = 509.52.

Question #324
Key: D

The retained loss is 100,000 + 0.15(100,000) + 0.10(100,000) + 0.05(150,000) = 132,500.

Question #325
Key: B

The expected losses for the primary insurer are 0.6(4,000,000) = 2,400,000. The expected proportion of losses in the treaty layer is (1.6/1.7 – 1/1.7 = 0.352941). The expected cost is 0.352941(2,400,000) = 847,058.

The relative cost of the layer can be derived using formulas from *Loss Models* as follows:

$$\frac{E(X \wedge 400,000) - E(X \wedge 100,000)}{E(X \wedge 500,000)}$$

$$= \frac{E(X \wedge 400,000)/E(X \wedge 100,000) - E(X \wedge 100,000)/E(X \wedge 100,000)}{E(X \wedge 500,000)/E(X \wedge 100,000)}$$

$$= \frac{ILF(400,000) - ILF(100,000)}{ILF(500,000)} = \frac{1.60 - 1.00}{1.70} = 0.352941$$

Question #326
Key: B

Total pure premium for 3 year experience = 752,000 / 1,875 = 401.

$$Z = \frac{1,875}{18,75 + 23,000} = 0.07538$$

Experience rating modification factor = 0.07538(401/475) + (1 – 0.07538) = 0.9883
CY4 experience rating premium = 380,000×0.9883×(1 – 0.10) = 337,999.

Question #327
Key: C

Information on losses below the deductible cannot be relied upon and hence none of the information from policies with a 500 deductible can be used. For the other policies, the total paid with a 100 deductible is (1,400 – 800) + (1,500 – 400) + (3,900 – 300) + (600 – 400) + (750 – 200) – (1,500 – 100) = 7,450, The total paid with a 200 deductible is 0 + (1,500 – 800) + (3,900 – 600) + 0 + (750 – 400) – (1,500 – 200) = 5,650. The relativity is 5,650/7,450 = 0.76.

Question #328
Key: E

Policies sold from November 1, CY5 to November 1, CY6 will be in effect through November 1, CY 7 and thus have an average accident date of November 1, CY6. For losses in AY4 the projection is 2.333 years and the projected cost is $1800e^{0.1275(2.333)} = 2423.58$. For losses in AY3 the projection is 3.333 years and the projected cost is $1550e^{0.1275(3.333)} = 2370.76$. The projected loss cost is the weighted average, (0.8)(2423.58) + (0.2)(2370.76) = 2413.02 = 2413.